USING REVENUE SHARING CONTRACT TO COORDINATE SUPPLY CHAIN WITH A RISK-AVERSE RETAILER

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ABSTRACT

Revenue sharing contract is an effective mechanism in supply chain coordination. However, whether or not it works when participants of the supply chain members have their own preferences on the risk attitude is an important research issue. This paper investigates this kind of issues. The retailer faces a risk preference measured by the exponential utility function and the supplier as a Stackelberg leader provides a revenue sharing contract for that retailer. The retailer then decides to place an order from the supplier according to his risk preference and random demand. We assume that the supplier known the retailer’s market information and its risk preference. Under this business situation, we find that the revenue sharing contract still can coordinate the supply chain in different contract parameters. However, the contract parameters can affect the supply chain total profit and the splitting of profit between the supplier and the retailer. The result of this paper can supplement the supply chain coordination research literature considering decision-maker’s risk attitude, and also be effective to support decision-maker to set incentive mechanisms or respond to the incentives.

Keywords-Supply Chain Risk Management; Revenue Sharing; Expected Utility Theory

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1 INTRODUCTION

Most previous studies of supply chain contracts assume the decision-makers are risk neutral (Eeckhoudt et al. [6], Tang [12] and Wu et al. [13]). In practice, however, they are usually risk-averse. In a dynamic and uncertain business environment, decision-makers are driven not only by their desire to maximize expected profits, but are also guided by the probability of (their decisions) achieving profit targets. If contracts that coordinate a channel incorporate only non-risk constraints, they may fail to coordinate effectively because the decision makers are risk-averse. In this paper, we analyze one kind of contract that frequently mentioned in literature and used in practice, that is, revenue sharing contract (see Cachon and Lariviere [3]). There are lots of literature about revenue sharing contract to coordinating supply chain (see Yao et al. [17] for review). In this paper, the supply chain coordination is defined following the Cachon [19] that the performance of supply chain member’s optimal actions by some kinds of coordination mechanisms leads to the equally performance as in the centralized channel, where the centralized channel means there is only one entity who makes the production and sale decisions. In the newsvendor model the action to coordinate supply chain is the retailer’s order quantity. When the retailers are risk-averse, whether and how the revenue sharing policy can be used to coordinate the supply chain has not been investigated. Therefore, additional research effort is needed to analyze and evaluate the revenue sharing contract to coordinate supply chain in theoretically and empirically to advance the industrial engineering knowledge concerning this important and expanding business strategy.

There are numerous studies on risk management in economics. Here we only focus on risk management issues in supply chain. A comprehensive literature review for supply chain risk can be found in the study of Tang[12]. An earlier paper that considered supply chain members risk is by Lau and Lau [10]. In Lau and Lau [10] study, the measure of supply chain risk is evaluated by the mean-variance model. Under the newsvendor supply chain structure, Lau and Lau [10] numerically show that the manufacturer’s (here supplier) returns policy can benefit the manufacturer himself but hurt the retailer, i.e. the so-called anti-intuition. Whether the contract obtained Pareto improvement depends on the manufacturer’s attitude towards risk. Under the same model structure as Lau and Lau [10], but with price-dependent demand, Agrawal and Seshadri [1] [2] adopt the increasing and concave utility function in profit to measure the supply chain member risk. They show how a risk-averse retailer chooses the order quantity and the selling price in a newsvendor inventory model. They consider two ways in which price affects distribution of demand: A change in standard deviation of distribution, and a change in only the mean value of distribution. They show that, in comparison to a risk-neutral retailer, a risk-averse retailer will charge a higher price and order less in the first case, while in case of the second scenarios, he will charge a lower price. Choi et al. [4] analyze the risk effect on the supply chain under a returns policy. They find that channel coordination is not always achievable under the risk controlled by mean-variance. This is sufficiently different with those most literature has reported that under ignoring risk aversions of the individual decision makers, channel coordination can always be achieved by setting a returns price.

Recently, Gan et al. [7] analyzed coordinated contracts (actually Pareto-optimal contracts) with three kinds of risk measures. They show that a wholesale contract can only reach a Pareto-optimal and the revenue sharing and buy-back contracts along with a side payment to the retailer can coordinate the supply chain under some conditions that satisfied the profit allocation proportion evaluated with risk measure. Later, Gan et al.[8] analyzed in detail one case in [7]. Specifically, they first analyzed the natural downside risk (NDR) of the buy-back and revenue-sharing contract, where the NDR is defined as the expected target profit level under risk-neutral newsvendor as the downside risk-averse newsvendor. Then they showed that the NDR with buy-back or revenue-sharing contract can not coordinate the supply chain. Therefore, they constructed a risk-sharing contract by which it can
coordination the supply chain with the wholesale price limited condition under retailer downside risk constraint. For the risk measured by the utility function, Gan et al. [7] modeled the supply chain with both supply chain agents are risk preference and found the Pareto-Optimal payoff set for both members. But for the specific coordinating contract, they designed the buy-back and revenue sharing contract to coordinate the supply chain. In our paper, we will investigate the supply chain coordination with the expected utility theory (EUT) to measure the risk the retailer faced only. Although the Wang et al [5] and Eeckhoudt et al. [6] have investigated the EUT for newsvendor’s pricing and ordering decision, they are not focus on the supply chain coordination. In fact, EUT has been widely used to measure the risk, whatever in financial or the project risk management.

2 THE ASSUMPTIONS AND CENTRALIZED CHANNEL

2.1 Model Assumptions

As a benchmark to study the supply chain performance, centralized channel consisted in only one decision entity will make the production/ordering quantity decision. The following notations will be used in whole paper.

- \( p \) --- the market price which is fixed by the market.
- \( x \) --- demand of market, random variable with \( f(x) \) and \( F(x) \) being PDF and CDF, respectively. Here assumes \( x \sim U[0, D] \), where \( D \) is the upper bound of a uniform distribution.
- \( w \) --- supplier’s wholesale price.
- \( c \) --- supplier’s production cost.
- \( \min\{x, Q\} \) --- take the smaller between \( x \) and \( Q \).
- \( Q \) --- ordering quantity or retailer order size from the supplier.
- \( \pi_{sc}, \pi_s, \pi_r \) --- supply chain profit, supplier profit, and retailer profit.
- \( CE \) --- supply chain efficiency, \( CE = (\pi_s + \pi_r)/\pi_{ic} \). Subscript \( ic, sc, s \) and \( r \) mean the centralized (also called integrated) channel, supply chain, supplier, and retailer. The * symbol stands for the optimal decision.

We follow the Wang et al [5] definition about the measurement of expected utility theory. Let the function \( U(W) \) be the newsvendor’s utility over his final wealth \( W \), where \( U(W) \) is twice differentiable. The following assumptions are hold for \( U(W) \) of the risk-averse newsvendor.

- A1. \( U'(W) > 0 \) for all \( W \),
- A2. \( U''(W) < 0 \) for all \( W \), and
- A3. \( \lim_{W \to \infty} U''(W) > 0 \).

Where, \( R(W) = \frac{-U''(W)}{U'(W)} \). Assumption A1 implies that \( U(W) \) is a strictly increasing with \( W \), which simply says that more wealth is desirable. A2 implies the diminishing marginal utility of wealth theory of risk aversion, i.e., a dollar that helps us avoid poverty is more valuable than a dollar that helps us become very rich. The assumptions A1 and A2 are common in the economics literature (see, e.g., [14], [15], and [16]). Assumption A3 is the well-known Arrow-Pratt measure of absolute risk aversion, which measures the insistence of an individual for more-than-fair bets ([14] and [15]).

2.2 Centralized Channel

With above assumptions, the centralized channel expected profit can be expressed by
\[ \pi_{ic}(x, Q) = pE[\min\{x, Q\}] - cQ \]  
where, \( \min\{x, Q\} = \int_{-\infty}^{\min(x, Q)} x f(x)dx + \int_{\min(x, Q)}^{\infty} Q f(x)dx = Q - \int_{0}^{Q} F(x)dx. \)

Simplifying (1), we have \( \pi_{ic}(x, Q) = (p-c)Q - p \int_{0}^{Q} F(x)dx. \)

To maximize the centralized decision maker’s utility, we have
\[
\max_{Q} E[U(\pi_{ic}(x, Q))] = \int_{0}^{h} U(\pi_{ic}(x, Q))f(x)dx
\]
where an \( h \) denotes the upper bound of \( x \).

According to assumption A1 and A2 of utility function, it is easy to show that
\[
d^2 E[U(\pi_{ic}(x, Q))]/dQ^2 < 0.
\]

By first order condition of (2), we have
\[
dE[U(\pi_{ic}(x, Q))]/dQ = (p-c-pF(Q))\int_{0}^{h} U(\pi_{ic}(x, Q))f(x)dx = 0
\]

Because of supply chain utility is not zero, we have \( p-c-pF(Q)=0. \)

That is
\[
Q_{ic}^* = F^{-1}\left(\frac{p-c}{p}\right)
\]

For our convenient to exposition, we use \( Q_{ic}^* \) to denote the optimal centralized channel order size. Under a uniform distribution of \( U(0, D) \), \( Q_{ic}^* = D(p-c)/p. \)

3 SUPPLY CHAIN CONTRACTS

3.1 Price-Only Contract

To evaluate the revenue sharing contract performance for coordinating the supply chain, a lot of literature often uses the price-only contract as a benchmark for evaluating the supply chain performance (Lariviere [11]). Here we assume that the retailer faces a risk preference measured by the EUT and the supplier provides a price-only contract for the retailer. The retailer can only take a take-it-or-leave-it action, that is, a Stackelberg follower (see Lariviere and Porteus [18]). With a random demand, the retailer’s decision model is as follows.
\[
\pi_{r}(x, Q) = \begin{cases} 
px - wQ & x \leq Q \\
pQ - wQ & x > Q 
\end{cases}
\]

With the risk measurement of EUT, the retailer’s decision will be
\[
E[U(\pi_{r}(x, Q))] = \int_{0}^{b} U(\pi_{r}(x, Q))f(x)dx + \int_{0}^{Q} U(\pi_{r}(x, Q))f(x)dx
\]
where, \( b \) is an amount that is less the infinite.
The first and second derivatives of expression (5) with respect to $Q$ are, respectively,
\[
dE[U(\pi, (X, Q))] / dQ = (p - w)^b U'(\pi, (X, Q)) f(x) dx - w \int_0^Q U'(\pi, (X, Q)) f(x) dx
\] (6)
and
\[
dE[U(\pi, (X, Q))] / dQ = (p - w)^2 \int_0^Q U''(\pi, (X, Q)) f(x) dx
- w^2 \int_0^Q U''(\pi, (X, Q)) f(x) dx - (p - w) U'(\pi, (X, Q)) \bar{F}(Q)
- w U'(\pi, (Q, Q)) \bar{F}(Q)
\] (7)

It is easy to see that (5) is less than zero (for proof, please see Wang et al [5]). The retailer has an optimal order with the following implicit function,
\[
(p - w) \int_0^Q U'(\pi, (X, Q^*)) f(x) dx - w \int_0^{Q^*} U'(\pi, (X, Q^*)) f(x) dx = 0,
\]
where $0 < Q^* < b$.

We assume the utility function is $U(x) = 1 - e^{-rx}$, $x > 0$, and $f(x)$--uniform under support $(0, D)$. Then (5) can become
\[
E[U(\pi, (X, Q))] = \int_0^Q [1 - e^{-r(p-w)Q}] f(x) dx + \int_0^Q [1 - e^{-r(p-w)Q}] f(x) dx
= 1 - \frac{D - Q}{D} e^{-r(p-w)Q} + \frac{e^{-r(p-w)Q} - e^{-rQ}}{rPD}
\] (8)
The first-order condition will be
\[
\frac{dE[U(\pi, (X, Q))]}{dQ} = \frac{e^{-r(Q-p)Q}}{PD} [rp(D - Q)(p - w) - w(e^{rpQ} - 1)] = 0
\]

Solve this equation, we have
\[
w = \frac{rp^2 (D - Q)}{rp(D - Q) + e^{rpQ} - 1}
\] (9)

Denote $G(w, Q) = rp(D - Q)(p - w) - w(e^{rpQ} - 1)$, we have
\[
\frac{dF}{dw} = -rp(p - w) - rpwe^{rpQ} < 0
\]
\[
\frac{dF}{dw} = -rp(D - Q) - (e^{rpQ} - 1) < 0
\]

Therefore, $\frac{dQ}{dw} = -\frac{F_w}{F_Q} < 0$. That is to say the order quantity deceases with the wholesale price. Now the supplier optimal decision model is as follows.
\[
\max_w E[\pi_s] = (w - c)Q
\] (10)

Substitute (9) into (11), we have
\[
E[\pi_s] = \left(\frac{rp^2 (D - Q)}{rp(D - Q) + e^{rpQ} - 1} - c\right)Q
\] (11)

If the retailer’s the optimal order equals the integrated channel order size, then the supply chain system will be gotten coordinated. Therefore, substitute (3) into (9), we have
\[ w = \frac{rp^2(D - Q)}{rp(D - Q) + e^{rpQ} - 1} = \frac{rpcD}{rcD + e^{rp(p-c)} - 1} \]

In price-only contract, it requests \( w > c \). So rearrange items in above equation one can get
\[ rD(p - c) + 1 > e^{rD(p-c)} \]. But this condition can never be satisfied. Therefore, with the risk-averse retailer, the price-only contract can not coordinate the supply chain. At the same time, analytically to solve (11) is intractable, so we will borrow the numerical study to explore the performance of price-only contract on coordinating supply chain. We leave it to Section 4.

3.2 Revenue Sharing Contract

Revenue Sharing Contract is operated in supply chain such as the supplier sells the good to the retailer with wholesale price \( w \) below the marginal cost of production, but needs to retailer to share \( \phi \) proportion profits back to the supplier (Cachon and Lariviere [3]). We further assume that the supplier and retailer know the knowledge about market demand, that is marketing information are common knowledge. Then we will show under the retailer face a risk constraint, under what conditions the whether the revenue sharing contract can coordinate supply chain and if it can coordinate the supply chain, what kinds of conditions be needed.

Suppose the retailer is of a risk-averse attitude, the supplier as a Stackelberg leader will set the revenue sharing contract conditions, that is, supplier provide for the retailer a wholesale price and revenue sharing proportion. The retailer, as a follower of Stackelberg game, will order the products from the supplier. We assume the product retail price is exogenous. For avoiding the retailer’s arbitrage, assume \( p > c > w \), \( 1 > \phi > 0 \), here \( \phi \) denotes the revenue proportion of retailer kept, and \( (1-\phi) \) will the share of supplier captured from the retailer revenue. Therefore, the supplier’s decision will be,
\[ \text{Max } E[\tau_w] = (w - c)Q + (1 - \phi)pE[\min\{x(Q), Q\}] \tag{12} \]

And the retailer’s optimal decision will be
\[ \text{Max } Q \quad E[U(\pi_r(x, Q))] = \int_0^P [1 - e^{-r(p\phi - w)Q} f(x)]dx + \int_0^Q [1 - e^{-r(p\phi - w)Q} f(x)]dx \]
\[ = 1 - \frac{D - Q}{D} e^{-r(p\phi - w)Q} + \frac{e^{-r(p\phi - w)Q} - e^{-wnQ}}{Dp\phi} \tag{13} \]

Where a \( D \) denotes a quantity less than infinite. The retailer’s first order condition of (13) can be
\[ \frac{dE[U(\pi_r(x, Q))]}{dQ} = \frac{e^{-r(p\phi - w)Q}}{\phi pD} [p\phi(D - Q)(p\phi - w) - w(e^{p\phi Q} - 1)] = 0. \]

Or \( p\phi(D - Q)(p\phi - w) - w(e^{p\phi Q} - 1) = 0 \). Solving \( w \) from this equation, we have
\[ w = \frac{(p\phi)^2 r(D - Q)}{p\phi r(D - Q) + e^{p\phi Q} - 1}. \tag{14} \]

At the same time, we get \( p\phi - w > 0 \), so \( \phi > w/p \).

Set \( F(Q, w, \phi) = p\phi(D - Q)(p\phi - w) - w(e^{p\phi Q} - 1) \), we have

34-6
\[
\frac{dF}{dQ} = -p\phi p(p\phi - w) - wrp\phi e^{r\phi - \phi} < 0 \\
\frac{dF}{dw} = -p\phi(D - Q) - (e^{r\phi - \phi} - 1) < 0
\]

Therefore, \( \frac{dQ}{dw} = -\frac{F_w}{F_q} < 0 \)

(15)

That is to say that the order quantity \( Q \) of the retailer is decreasing with wholesale price \( w \) of the supplier.

Substitute (14) into (12), one can get the supplier’s optimal expected profit, in which it is

\[
E[\pi_s] = \left( \frac{(p\phi)^2 r(D - Q)}{p\phi(D - Q) + e^{r\phi - \phi} - 1} - c \right) Q + (1 - \phi) p \min\{X, Q\} \\
= \left( \frac{(p\phi)^2 r(D - Q)}{p\phi(D - Q) + e^{r\phi - \phi} - 1} - c \right) Q + (1 - \phi) p \left( Q - \int_0^Q F(x)dx \right) \\
= \left( \frac{(p\phi)^2 r(D - Q)}{p\phi(D - Q) + e^{r\phi - \phi} - 1} - c \right) Q + (1 - \phi) p \frac{2DQ - Q^2}{2D}
\]

(16)

Suppose the coordination will be obtained if the supplier’s revenue sharing contract can make the retailer’s order quantity under decentralized decision reaching to the centralized situation, that is, \( Q^* = Q_{ic}^* = \frac{(p - c) D}{p} \), substituting it into (16), we have

\[
w = \frac{p\phi^2 rcD}{\phi rcD + e^{r(p-c)D\phi} - 1}
\]

(17)

In revenue sharing contract, because the \( w < c \) is allowed, the (17) can be satisfied in most situations. That is to say, the supplier’s optimal decision, in fact, is a function of the revenue sharing proportion \( \phi \). We can say that if the supplier set the any pairs of \( (w, \phi) \) that satisfy (17), the supply chain will be coordinated. At the same time, if the retailer’s expected profit in the decentralized channel is not below the centralized situation, he/she willings to participate the revenue sharing mechanism with the supplier. As in the price-only contract, analytically solve the optimal problem (16) is intractable, so we still use numerical study to insight the management implication.

4 NUMERICAL STUDY

Analytically solving the above problems seems intractable, so in this section, we borrow the numerical method to solve the above problems to get optimal decisions and then analyze the contract properties and the impact of the risk on the supply chain performance. Here we introduce the parameters used in numerical study. As is well known, the coefficient \( r \) in exponential utility function can be used to measure the risk attitude. The bigger of the \( r \) value is, the more averse risk for the decision maker. In our numerical study, we assume \( r = 0.0001, 0.001 \) and 0.01. When \( r = 0.01 \), the retailer will be very risk averse, and vice versa. The retail price will be exogenous (decided by the market), here \( p = 3, 4, 5, 6, 7 \) and 8. In addition, the random demand will follow the uniform distribution in \([0, 100]\) and production cost \( c \) will be equal 1. The results are exhibited in Table A1 and A2 of Appendix.

Due the focus of this paper mainly on the revenue sharing contract, in the following analysis we concern more on the Table A2.
(1) The CE=100% in Table A2 shows that decentralized channel optimal profits equals the centralized channel optimal profits. That is to say, given any pair of retail price \( p \) and the risk averse measure value \( r \), there can always find one pair of \((w, \phi)\) to make the revenue sharing contract coordinating the supply. From theoretical perspective, if the supplier’s contract set the \((w, \phi)\) satisfying (9), the supply chain can obtain coordination. On the other hand, with the increase of the retailer’s risk-averse attitude, in each optimal pair of \((w, \phi)\), the \( \phi \) decrease will lead to the decrease of \( w \), but the supply chain profit will be fixed. However, the retailer’s profit will decrease and the supplier’s profit will increase. However, CE<1 in Table 1 has shown that price-only contract cannot coordinate the supply chain.

(2) Under retailer risk-averse attitude, the revenue sharing contract can inflexibly to allocate the profits between the supplier and retailer. Because each pair of \((w, \phi)\) combination to satisfy (6) is fixed under optimal decision. As a Stackelberg leader the supplier may get more profits, if only the retailer’s profit is not below the price-only contract profit. However, doing so by the supplier, the retailer’s share will be reduced much more, and the supply chain may not be coordinated. Therefore, the supplier should use this contract with reasonable scope so that the risk-averse retailer positively participate the business.

(3) In some marketing condition, the wholesale price and revenue sharing proportion can affect the profit of the retailer and the supplier. Specifically, with the wholesale price and revenue sharing proportion decrease, the retailer’s share will be decrease, but the supplier’s share increase. From Table A1, for example, when \( p =3 \), the revenue sharing proportion \( \phi \) decreases from 0.2563 to 0.2492, the retailer’s profit decreases from 18.4654 to 16.6387, but the supplier’s profit increases from 48.2013 to 50.0280.

(4) The supplier should choice the reasonable wholesale price and revenue sharing proportion to make the business trading continuation. Because the each pair of optimal \( w \) and \( \phi \) makes the profit allocation not fixed, that is, \( \pi_r/\pi_s \) is different with different \((w, \phi)\).

(5) With a fixed risk measure value \( r \), the increase of retail price will lead to the supply chain, the retailer and the supply profits increase significantly. The ratio of retailer’s profit with the supplier’s profit \( \pi_r/\pi_s \) be changed. Specifically, with the retail price increases, the increase of retailer risk attitude (risk measure value \( r \) increase) will lead to the retailer’s share decrease. This observation seems certified one buzzword that the greater the risk, the greater gains.

5 CONCLUSIONS AND FUTURE STUDY

In this paper, we model a supply chain with the revenue sharing contract under the retailer risk averse. Assume the market information is common knowledge, we use the exponential utility function to model the retailer’s risk attitude and with the newsvendor model to build the supply chain profit and use the Stackelberg game to play the business operation. We first analyze theoretically the coordinating condition of revenue sharing contract, that is, if the supplier design a pair of contract parameter \((w, \phi)\) to satisfy \( w = \frac{p \phi^2 rcD}{\phi cD + e^{(p-c)D \phi}} - 1 \), then the supply chain will be coordinated. In fact, given the risk attitude and retail price, the optimal pair of \((w, \phi)\) are fixed in coordination conditions. Then we borrow the numerical methods to analyze the effect of the risk attitude, retail price and contract parameters on the supply chain profit and allocation of profit between the retailer and the supplier and so on. Results show that as in the no risk constraint condition, the revenue sharing contract under retailer risk-averse attitude can still coordinate the supply chain. Compared with that price-only contract, the revenue sharing contract can effectively improve the total supply
chain profits. More interestingly, the higher the retailer’s risk-averse attitude is, the lower the supplier’s wholesale price will be and the lower of the retailer profits share will be. Of course, the supplier’s share will be high due to the bearing the risk. There is a limit to analyze the revenue sharing contract for coordinating the supply chain in this paper, that is, we omit the cost of the supplier to monitor the retailer’s revenue. Retailer may hide their revenue so that the supplier share will be loss. Therefore, in future study, a more practice model should include the monitor cost with the supplier profit. However, with the modern information technology development, this can be easy taken in technique.

Acknowledgement:

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Appendix

Table A1: The Numerical Optimal Solution With Price-Only Contract Under Uniform Distribution [0,100]

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<th>p</th>
<th>r</th>
<th>Q*</th>
<th>(\pi_r)</th>
<th>w</th>
<th>(\pi_s)</th>
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<th>(\pi_r+\pi_s)</th>
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Table A2: The Numerical Optimal Solution With Revenue-Sharing Contract Under Uniform Distribution [0,100]

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6 REFERENCES


