Feature Based Reverse Engineering of Compressor Blades

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Abstract
Reverse engineering has progressed in significant ways in the decades since Pierre Beziér first started with what is arguably the first modern reverse engineering process in industry. Modelling philosophies have progressed from reconstructing surface patches to full blown parametric feature based reverse engineering. This paper will investigate the latest modelling approach on a compressor impeller, ultimately for CFD analysis. Small features on parts, such as the tip radius of a compressor impeller blade, can be significant for the performance of the product. The blade consists of three functional surfaces, the pressure, suction and tip surfaces. The tip radius was less than 1 mm, making it difficult to scan points on its surface and separating it from points on the pressure and suction surfaces. Instead, a constrained optimisation strategy was employed to simultaneously approximate the surfaces as features and find the tip radius without actual measurements on it. The strategy is presented with some accuracy results. It was found that the tip surface could be modelled in this way.

Keywords
Reverse Engineering, Features, Computer Aided Design, Compressor Blades

1 INTRODUCTION
The impeller in Figure 1 was modelled from a physical part. The purpose of the model was to do a CFD analysis as part of a research project. The tip radius, as all the rest of the geometry of the functional surfaces, is critical for the performance of the compressor; therefore an accurate representation was needed. Unfortunately, the small size of the tip surface (tip radius < 0.5 mm) makes it very difficult to measure it. Also, if a full scan of the entire blade is made, it will be hard to separate the tip points from the points on the pressure and suction surfaces, because of the tangential continuity between the surfaces. Furthermore, it was discovered that the tip surface is slightly conical and its centreline is not perfectly radial to the impeller hub. All the aforementioned complicates measuring accurate radii on the tip surface.

Figure 1 - Impeller

It was decided to forgo having points on the tip radius. Instead, the pressure and suction surfaces where approximated with constraints that fixed the tip surface as well.

Initial approximations showed that additional control of the surfaces next to the tip was necessary. There was not enough scanned data in this region to sufficiently control the shape. The problem was a slight S-bend just after the tip, see Figure 2. Although the fitting accuracy was sufficient, this bend could cause problems for the CFD simulation as it may disrupt the flow over the blade. As a result, curvature constraints were added to the model.

This case study is similar to another on the internet [1] in that a feature based modelling approach is used. However, in the mentioned case study, the blade surfaces, including the tip is modelled with free form NURBS surfaces. The current case study goes a step further in that the blade is subdivided into two free form B-spline surfaces and a conical tip surface. Furthermore the relationship between these surfaces is controlled automatically during the surface approximation phase.

Figure 2 - This figure shows the slight S-bend in both blades - note the edges - after the first trials.

In feature based reverse engineering (FBRE) the constraints between surfaces are important. While this case study was successful, the algorithm presented here is not a truly generic FBRE system, but the case study highlights some of the problems that must be addressed in a FBRE system. Some other recent work on FBRE systems include the
work on identifying features in point sets [2], a commercial system implementing alternative approaches to FBRE [3], extracting sharp edges from a point set [4], extracting datum features for tolerance control [5] and simultaneous constrained fitting and integration of multiple point sets [6].

The paper is structured as follows. First the geometric model is described. This is followed by a section on the approximation and finally the results. The paper ends with conclusions.

2 MODELLING

The surface model is shown in Figure 3. The suction and pressure surfaces were modelled with cubic ruled B-spline surfaces (i.e. degree 1 in the rule direction and degree 3 in the other direction). In Figure 3 the suction surface’s control points are indicated with the letter \( S \) and the pressure side’s with the letter \( P \). The parameter space \((u,v \in 0..1)\) on the pressure surface is such that \( u,v=0 \) at point \( P_{1,1} \) and \( v=1 \) at \( P_{1,2} \) and on the suction surface \( u,v=0 \) at point \( S_{1,1} \) and \( v=1 \) at \( S_{1,2} \). The surfaces meet the tip surface tangentially.

\[
\begin{align*}
\theta_1 & = \theta_2 = \theta_3 = \theta_4 = \frac{\pi}{2} \\
\theta_1 & = \theta_2 = \theta_3 = \theta_4 = \frac{3\pi}{2}
\end{align*}
\]

The impeller centreline is on the Z-axis. The highest point along this axis on the blade was measured and used as input, called \( z_{\text{max}} \) below. The large and small radii of the tip surface cone are \( r_1 \) and \( r_2 \) and centre points are \( t_1 \) and \( t_2 \) respectively. Now,

\[
t_{1z} = t_{2z} = z_{\text{max}} \cdot r_1
\]

where \( t_{1z} \) indicates the z-component of the point \( t_1 \). All points are considered column vectors.

Let \( d \) be the unit vector from \( t_1 \) to \( t_2 \) and let \( R(\theta) \) be a rotation matrix about the axis \( d \), with \( \theta \) being the rotation angle. Then the tangential boundary points of the pressure side surface are

\[
P_{1,1} \hat{=} R(\theta)(t_1 + (0, 0, t_{1z} + r_1))
\]

and

\[
P_{1,2} \hat{=} R(\theta)(t_2 + (0, 0, t_{2z} + r_2))
\]

The tangency is ensured by specifying the position of the second row of B-spline control points \[7\]. The tangency direction, \( P_t \), is found from the following vector product.

\[
P_t = (P_{1,1} \times t_1) \times d
\]

Then the control points in the second row are

\[
P_{2,1} = P_{1,1} + \beta_1 P_t
\]

and

\[
P_{2,2} = P_{1,2} + \beta_2 P_t
\]

Each B-spline surface has 16 control points (and 12 for the smaller blade) including the ones fixed by the above mentioned constraint equations. The equations for the suction side control points are done in the same way. These points as well as the model parameters \( \theta_{\text{pressure}}, r_1, r_2, t_{1x}, t_{1y}, t_{2x}, t_{2y}, \beta_1, \beta_2 \) and similar parameters for the suction surface must be determined through a non-linear least squares minimisation.

The angle \( \theta \) was constrained as follows for the pressure surface

\[
\frac{\pi}{2} \leq \theta_{\text{pressure}} \leq \frac{3\pi}{2}
\]

and

\[
\frac{-\pi}{2} \leq \theta_{\text{suction}} \leq 0
\]

for the suction surface to ensure a feasible solution.

The negative curvature was avoided by imposing a constraint on the control polygon immediately after the edge with the tip surface. To derive the constraint, define \( n_{ij} = (P_{i+1,j} - P_{i,j}) \times (P_{i,j+1} - P_{i,j}) \) as the normal vector to the control polygon at point \( P_{i,j} \). The constraint was derived by observing that for the downward slope of the blade (note, not all the way to the end of the blade) the scalar product of \( k_{ij} \), \( d \geq 0 \), where \( k_{ij} = n_{ij} \cdot P_{i,j+1} - P_{i,j} \) and the pressure side’s normal vector \( n_{ij} \). In the initial trials, the S-bend was caused by the fact that at \( P_{2,1} \) and \( P_{2,2} \), \( k_{ij} = 0 \), and \( k_{ij} < 0 \), see Figure 4.

The partial control polygon on the left (a) shows the ideal case that will ensure a correct transition from the tip surface to the blade. On the right (b) a control polygon is shown as was found in the first trials. It caused the S-bend in the surface.

For the curvature constraint, the following normal vectors to the control polygon was first determined

\[
n_{1,1} = (P_{1,2} - P_{1,1}) \times (P_{2,1} - P_{1,1})
\]
\[ n_{1,2} = (P_{2,2} - P_{2,1}) \times (P_{3,1} - P_{2,1}) \quad (10) \]
\[ n_{2,1} = (P_{1,2} - P_{1,1}) \times (P_{2,2} - P_{1,2}) \quad (11) \]
\[ n_{2,2} = (P_{2,2} - P_{2,1}) \times (P_{3,2} - P_{2,2}) \quad (12) \]
The cross product of the normal vectors gives \( k \).
\[ k_{1,2} = n_{1,2} \times n_{2,1} \quad (13) \]
\[ k_{2,2} = n_{2,2} \times n_{2,1} \quad (14) \]
For no negative curvature, these constraints apply:
\[ k_{1,2} \cdot d \geq 0 \quad (15) \]
\[ k_{2,2} \cdot d \geq 0 \quad (16) \]
This ensures that \( k_{1,2} \) and \( k_{2,2} \) are both in the same direction as \( d \).

Furthermore, to ensure that the tip diameter is not more than the blade thickness immediately after the tip, the tip surface angles were constrained as
\[ \pi - (|\theta_{\text{pressure}}| + |\theta_{\text{suction}}|) \geq 0 \quad (17) \]
The cone base and top radii were constrained as
\[ r_1 \geq r_2 \quad (18) \]
This was done, because \( r_2 \) was marginally larger than \( r_1 \) during the initial trials for the small blade. The constraint forced the two radii of the small blade to be equal after the optimisation. This agreed with vernier measurements of the small blade tip surface.

3 MEASUREMENTS

The blades were measured with a coordinate measurement machine (CMM) with a 3 \( \mu \)m volumetric measurement uncertainty. There were 147 points scanned on the suction side and 150 on the pressure side (of which only 141 was ultimately used) (137 and 148 (using only 140) respectively on the smaller blade). The probe diameter was 0.968 mm. No automatic probe radius compensation was done. A more accurate offset could be done during the fitting process by using the base surfaces to find the offset direction. The measurements took about 2.5 hours.

4 APPROXIMATION

If
\[ S(u, v) = \sum_{i=1}^{n} \sum_{j=1}^{m} B_{i,p}(u) B_{j,q}(v) P_{i,j} \quad (19) \]
is a general B-spline surface (following the notation of [7] where \( B_{i,p}(u) \) is the \( p \)th B-spline basis function of degree \( p \), \( P_{i,j} \) is the \( (i,j) \)th control point and \( u, v \) is the surface parameters), then
\[ \sum_{i=1}^{N} ||Q_i - S(u_i, v_i)||^2 \quad (20) \]
must be minimised, where \( Q \) is the \( i \)th scanned point, \( N \) is the number of scanned points and \( (u_i, v_i) \) are the parameters on \( S(u, v) \) closest to \( Q \).

The base surface parameterisation method [8] was used. A rough approximation of the surface was made manually through the uncompensated points in a CAD package. Then a cubic ruled B-spline surface was fitted to the uncompensated points. This surface was used for the first point offset calculation. After the offset, the surface was refitted to the compensated points. This surface was used as an initial value for the SQP (sequential quadratic programming) algorithm [9].

Next, the algorithm went into a loop that first recalculated the offset using the latest surface approximation and then did the SQP optimisation. After the SQP algorithm, \((u_i, v_i)\) was recalculated for the next loop cycle. The main purpose of this loop was to improve the offset calculation. This improved the maximum absolute error from 0.569 mm to 0.176 mm in two iterations.

This problem was sensitive to the initial values of the optimisation algorithm, leading to the conclusion that there are many local minima. The best results (yet not guaranteeing a global minimum) were obtained by using the first approximation of the surface as initial values for the control points. The other parameters were chosen as follows from the control points of the initial surface
\[ t_{1x} = (P_{1,1x} + S_{1,1})/2 \quad (21) \]
\[ t_{1y} = (P_{1,1y} + S_{1,1})/2 \quad (22) \]
\[ t_{2x} = (P_{1,2x} + S_{1,2})/2 \quad (23) \]
\[ t_{2y} = (P_{1,2y} + S_{1,2})/2 \quad (24) \]
\[ r_1 = 0.6 \quad (25) \]
\[ r_2 = 0.35 \quad (26) \]
\[ \theta_{\text{pressure}} = \frac{2\pi}{3} \quad (27) \]
\[ \theta_{\text{suction}} = \frac{\pi}{6} \quad (28) \]
\[ \beta_{1,\text{pressure}} = \|P_{1,1} - P_{2,1}\| \quad (29) \]
\[ \beta_{2,\text{pressure}} = \|P_{1,2} - P_{2,2}\| \quad (30) \]
\[ \beta_{1,\text{suction}} = \|S_{1,1} - S_{2,1}\| \quad (31) \]
\[ \beta_{2,\text{suction}} = \|S_{1,2} - S_{2,2}\| \quad (32) \]

It was also found that the best approximation of the tip surface's cone axis direction was obtained if a weighted constraint function was used. The errors of the points on the neighbouring surfaces closest to the tip surface were weighted higher in the following way:
\[ w_i = 1.1 \text{ if } u < 0.2 \text{ otherwise } w_i = 1 \text{ where } w_i \text{ is the weight associated with } Q_i \text{ and } u_i \text{ is the surface parameter associated with it.} \]

5 RESULTS

The results of the surface fitting are summarised in Table 1. The tip radii values compare well with blade thickness measurements done with a vernier at the tip. The vernier measured 1.1 mm and 0.8 mm respectively for \( r_1 \) and \( r_2 \). The errors are between -0.131 mm and 0.176 mm which can be considered a
sufficiently accurate result, bearing in mind that the original blade is a casting where manufacturing errors in this range are common.

During the approximation it was found on the pressure side that some points that were scanned very close to the boundary between the tip and pressure surface caused problems for the algorithm. The boundary obviously moved between iteration steps and after some steps these points where no longer on the pressure surface. This algorithm has no function to eliminate such points automatically. During some trials these points ended up over the tip surface and resulted in a degraded approximation of the tip radii. Ultimately the points were manually removed from the point set and this resulted in a more stable solution. The importance of segmenting the point set correctly in FBRE systems have already been identified by [2].

<table>
<thead>
<tr>
<th></th>
<th>Large Blade</th>
<th>Small Blade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Error</td>
<td>-0.131 mm</td>
<td>-0.110 mm</td>
</tr>
<tr>
<td>Maximum Error</td>
<td>0.176 mm</td>
<td>0.154 mm</td>
</tr>
<tr>
<td>Average Error</td>
<td>0.013 mm</td>
<td>0.004 mm</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.053 mm</td>
<td>0.045 mm</td>
</tr>
<tr>
<td>Tip Radius r1</td>
<td>0.498 mm</td>
<td>0.330 mm</td>
</tr>
<tr>
<td>Tip Radius r2</td>
<td>0.408 mm</td>
<td>0.330 mm</td>
</tr>
</tbody>
</table>

Table 1 - Surface Fitting Results (Results for the larger and smaller blade are shown, although only the modelling of the larger blade is discussed here. The smaller blade was modelled in a similar fashion.)

6 CONCLUSIONS

This study showed that an accurate estimate of tip radii of an impeller blade could be made without actually measuring points on its surface, using a FBRE approach. The suction and pressure surfaces were scanned and the data were approximated with appropriate B-spline surfaces. Constraints were added to ensure proper integration of the tip surface with the rest of the model. The rest of the tip surface parameters were determined as part of the optimisation process while approximating the suction and pressure surfaces.

It was difficult to determine beforehand what the model should be. Two aspects of the tip surface that was discovered only after several trials were that it was actually a conical surface and that its rotation axis was not radial. Once the model was adapted to accommodate these geometry changes, its accuracy significantly improved. It must also be noted that the objective of this study was to replicate the physical blade accurately; therefore the original design intention was not considered. It is therefore not relevant, and also not known, whether the original design intended this geometric model.

It was also critical that the surface parameters \((u, v)\) be optimised as well. This significantly improved the accuracy, but it came at the cost of computational time, since it significantly increased the parameter space for the optimisation algorithm.

Using a weighted objective function also helped to get the tip's conical axis direction right. The points near the tip surface were weighted slightly higher than the rest of the points.

Finally, as explained in the results section, the points had to be carefully selected so that all points lie on the surface being fitting during all steps of the algorithm.

7 REFERENCES


8 BIOGRAPHY

Kristiaan Schreve obtained his PhD degree in Mechanical Engineering from the University of Stellenbosch in 2002. Since 2004 he is a full time lecturer at the same university. His research interests include reverse engineering and vision based dimensional metrology.