OPTIMIZING INVENTORY ORDERING POLICIES WITH RANDOM LEAD TIMES

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ABSTRACT

One area of focus by inventory managers is the ability to cope with random fluctuations in lead times. These random lead times may be caused by many factors such as run-time or operation time, set-up time, waiting time, customers’ demands, machine down-times, handling time and lot size inspection time which may all be random. The present paper concentrates on the impacts of maintenance operations and fluctuation in demand volume on the random lead times that in turn affect the optimal inventory level. The paper proposes stochastic models for periodic (R,S) and continuous (r,q) inventory levels that depend on total Lead Time. Results reveal that increase in machine breakdown time brings about a reduction in depletion rate and increase in demand increases the depletion rate. An increase in depletion rate is revealed by the fact that time evolution of the inventory level lies above that of a deterministic system. Thus the optimal inventory level can only be determined by following the plot derived from the model.

Keywords: random demand, random maintenance operation, random Lead Time, inventory level
1 INTRODUCTION

Lead Time is defined as the time that elapses between the placement of an order and the receipt of the order. It is usually defined for a stock-pile-up inventory or stock-shipment inventory. In most deterministic and stochastic inventory models encountered in the literature, the optimal policy is determined with the assumption that Lead Time is independent parameter [1]. However, Lead Time is composed of many controllable components such as run time, set-up time, waiting time, moving time and lot size inspection time [1]. Lead Time may influence customer service and impact inventory costs. These effects of Lead Time are well known but are too general to be used in practical ways. In fact, under practical situation, Lead Time should be reduced. Consequently, it is important to know how and to what extent each of the many components of the manufacturing lead time influence the level of inventory in order to select the most cost effective inventory model [2].

There is a growing literature on modelling the effects of changing Lead Times on inventory control models. Many studies that deal with Lead Time reduction in inventory review models have been performed in the past years [3,4]. Lin [5]. Examined the effects of the reduction of the Lead Time associated with the controllable backorder rate in the periodic review inventory. Later, many methods for reducing Lead Time and their impact on the safety stock and the expected total cost of a continuous (r, q) inventory review models were studied [6]. The focus by Glock [6] was on a single vendor- single buyer integrated inventory model with stochastic demand and variable lot size-dependent Lead Time. It was assumed that lead time consist of production, set-up and transportation times. As a consequence, it was found that Lead Time may be reduced by increasing producing rate or by reducing the lot size [6]. In many practical ways, by reducing the Lead Time, it is possible to lower the safety stock, reduce the stock-out level and improve the customer service level. It should be noted that the work of Glock [6] was dealing with random demand (or specifically variable lot size). Furthermore, the work did not include the impact of maintenance operations that may take place. It should, further, be noted that the variable lot size as proposed by Glock [6] can be deterministic and not stochastic as considered in the present paper. Thus, this present paper seeks to improve on previous results by considering random downtimes (or maintenance) and random demands (or stochastic lot size). It should be emphasized that the inventory level at any time depends on the actual inventory policy in place.

In classical periodic (R,S) and continuous (r,q) inventory review models with deterministic demand and Lead Time, the problem of shortage is easy to address. It is possible to predict what the inventory level would be when an order arrives. This is not easy when the demand and Lead Time are random. For instance, there are situations where the replenishment may take even longer. This situation happens if there are disruptions at the producer level. That is why it is necessary to carry some additional stock to avoid stock-out costs that may occur, especially when the Lead Time is also random. This stock, known as safety stock, is defined as the expected value of the net inventory at the time an order arrives. In reality, when the fluctuations in demand are high, a reasonable amount of safety stocks are required to avoid stock-out, and as result, holding costs are increased. This issue indicates that controlling the Lead Time should be the principal concern in backordered environment because it directly affects the safety stock level. Unfortunately, for classical periodic (R,S) and continuous (r,q) inventory review models, the lead time and safety stock are rarely designed to be fully utilized under different and uncertain conditions such as poor operation maintenance performance and demand fluctuation. A sound mathematical approach to Lead Time will therefore help to determine the appropriate inventory level.
2 METHOD

The relationship between inventory level, flow rate and time that may be used for inventory system is represented as follows [7].

\[ I = I_0 \pm \frac{Q}{MLT} t \]  

(1)

Where \( I_0 \) is the initial inventory, \( Q \) is the demand, \( MLT \) is the manufacturing Lead Time or inventory replenishment time, \( t \) is the time and \( Q/MLT \) is the flow rate. The “±” is used to represent whether stocks are piled up (+) or shipped out to customers (-).

The ability of manufacturing firm to keep the optimal product inventory level is hugely dependent on demand. In general, it is assumed that the \( MLT \) model comprises at least demand size (or lot size), the waiting time, processing time, inspection time, moving time and queuing time, which are all stochastic in nature. The general mathematical formulation of the manufacturing Lead Time can be written as follows [7].

\[ MLT(t) = n_0 (T_{su} + QT_c + T_{no}) \]  

(2)

Where \( MLT \) = total lead time for an order, \( Q \) = Batch quantity of all product or demand or lot size, \( n_0 \) = Number of “machines” along the line, \( T_{su} \) = up time or waiting time to prepare for the batch quantity \( Q \), \( T_c \) = Operation cycle time and \( T_{no} \) = Non operation time (source of delay mainly due to transportation time, waiting time, queuing time, etc.). From the general model, the \( MLT \) or delivery cycle time that is dependent on the type of maintenance program implemented is therefore determined.

2.1 MLT with reactive maintenance time

Reactive maintenance is the type of maintenance in which equipment is maintained when they break down or which occurs within production cycle. This mode of maintenance is still being preferred and employed by some African manufacturing companies since it requires less staff and does not incur capital cost until something breaks. In this case, the \( MLT \) may be represented by the following formula:

\[ MLT(t) = n_0 \left( T_{su} + QT_{operation} + F_{Corrective} * T_{Corrective} + T_{no} \right) \]  

(3)

2.2 MLT with preventive maintenance time

Preventive maintenance program is the type of operation in which actions are performed according to a specific schedule so as to detect, prelude or minimize the degradation of equipment, or extend their life through controlling their degradation to an acceptable level. Since preventive operation is performed while equipment is at rest, the manufacturing Lead Time is given as:

\[ MLT(t) = n_0 \left( T_{su} + QT_{operation} + F_{preventive} * T_{preventive} + T_{no} \right) \]  

(4)

2.3 MLT with predictive maintenance time

Predictive maintenance is the type of operation in which measurements that detect the beginning of equipment degradation are taken so that they can be eliminated or controlled in either reactive or preventive maintenance. In predictive maintenance, the \( MLT \) is, thus, given as follow:
\[ \text{MLT}(t) = N_0 \left\{ T_{SU} + Q(T_{\text{operation}} + F_{\text{corrective}} T_{\text{corrective}}) + F_{\text{preventive}} T_{\text{preventive}} + T_{m0} \right\} \] (5)

For all the three types of maintenance operations mentioned above, the probability of breakdown or downtime is represented by \( F_i \) and the average downtime is represented by \( T_i \) where \( i \) stand for the type of maintenance operation. The time increment of MLT may then be obtained and results made stochastic by addition of periodic and continuous fluctuation terms given as.

\[ d(\text{MLT}(t)) = T_{C} \frac{\partial Q}{\partial t} dt + \text{Fluctuation} \] (6)

Equation (1) and (6) are solved simultaneously while taking into consideration the constraints given by expression (3), (4) and (5)

3 RESULT AND DISCUSSION: Illustrative examples

An illustrative simulation-optimization for the three different maintenance programs is dealt with. The parameters to be dealt with are obtained from empirical data. The expected demand may evolve linearly. Due to the continuous and periodic fluctuations, the demand quantity can be represented as follow as

\[ q = a + \int_0^t (b + \int_c + \int_p) dt \quad \text{where} \quad \int_c = c \text{t}, \int_p = p \sin (dt) \quad a=20= \text{initial demand}, \ b=0.025= \text{rate of change of demand}, \ c=0.00007, \ d=1, \ p=4. \]

The machine parameters are given as \( n_0=3, \ T_{SU}=1.5\text{min}, \ T_{m0}=0.5\text{min}, \ T_{\text{operation}}=1.5\text{min}, \ F_{\text{corrective}}=0.0008, \ F_{\text{predictive}}=0.000008, \ F_{\text{preventive}}=0.009, \ T_{D\text{corrective}}=2\text{min}, \ T_{D\text{predictive}}=1.5\text{min}, \ T_{DP\text{preventive}}=0.5\text{min} \text{ and } T=6000\text{min}. \) It should be noted that \( f_c \) and \( f_p \) are fluctuation terms respectively. The results are presented in the plots below.

![Figure 1: The impact of maintenance program time and demand on the MLT an](image)

It should be observed that the demand volume fluctuates in the same way as manufacturing lead time, although MLT increases more rapidly than demand. This rapid increase is because the MLT is the time that it takes from the receipt of demand-order until when the demand is completely delivered to the satisfied customer. The deviation between the MLT and demand increases as time progresses is shown in figure 1 and figure 2. The higher increase (or deviation) of the MLT may be due to the fact that equipments may break down during
the demand-lead time, and it takes random amount of time maintaining them. The time spent maintaining these equipments should then increase the MLT and may therefore affect the delivery of products to customers. In fact the long run behaviour shows that demand evolves linearly while the MTL evolves in a parabolic manner which is not one-to-one relationship as many be predicted by expression (2).

The effects of MLT and demand level on the inventory level are found in figure 2. It should be recalled or mentioned here that the inventory trend or inventory level depends on whether inventories are piled up or shipped out. From shipment operation angle, it is observed that a linear increase in demand results in a parabolic decrease in the inventory level. It is obvious that an increase in demand should bring about a decrease in inventory level for shipment operations, but the interesting revelation is that this decrease in inventory level is not linear as that of demand. The long run behaviour of the shipment operation indicates a point whereby there is rapid increase in inventory level as demand increases before another decay with further demand increase. This abrupt increase is a subject of further investigations. For inventory pile-up, it is seen that as demand increases linearly, the inventory level decreases in a parabolic manner, but this time with the axis of symmetry being the time axis. The abrupt drop in demand is also observed under this scenario.

One can then see that the relationships between demand level, maintenance time and inventory level are not one-to-one. Thus, the use of the models proposed here might be beneficiary.

4 CONCLUSIONS AND RECOMMENDATION
This paper shows how simulation model and simulation optimization can be used to investigate the effects of maintenance program time and varying demand volume on the inventory level which in turn affects the manufacturing system performance.

It was also found that as time goes by, maintenance operation affects the MLT and then the inventory level depending on whether the analysis is made on stock pile-up or stock shipmet angle.
It is also observed although demand may vary linearly, the variations in MLT and inventory levels turned out to be parabolic, which does not trivially follow up from the one-one-relationship as give by expression (2).

Thus, the use of the models proposed here might be beneficiary as they indicate deviations that are inherent from the probable maintenance operations.

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5 REFERENCES