SAIIE29 Proceedings, 24th – 26th of October 2018, Spier, Stellenbosch, South Africa © 2018 SAIIE

TAYLOR SERIES METHOD FOR THE ANALYSIS OF NON-LINEAR AND DYNAMIC ENERGY CONSUMPTION MODEL UNDER A FLEXIBLE INVENTORY MODEL

B.B. Benga[1], T.B. Tengen[2] & A. Alugongo[3]

[123]Faculty of Engineering and Technology,
Department of Industrial and Mechanical Engineering,
Vaal University of Technology
[1]bengaebouele@yahoo.fr, [2*]thomas@vut.ac.za, [3*]alfayoa@vut.ac.za

ABSTRACT

Existing literature shows that operational inefficiency is one important cause of energy consumption problem encountered within most manufacturing plants. Energy consumption can be perceived as an important key performance indicator because it is mostly influenced by the production rate, which in turn is linked to the lot-size and lead-time. Machines and processes within a plant produce goods at variable rates. When speeds vary, slow rates typically result in dropped profits while faster speeds affect quality control. Producing more, earlier, or faster than required by the next process is waste. It causes inventory to accommodate the excess and frequent reprocessing. This is why it is important for operating speeds to remain consistent with the inventory model's parameters, and it can be done by carefully handling the random behavior of lot-size and lead-time. In this research paper, Taylor Series Method is used to analyze, approximate the dynamic property of energy consumption around the average lead-time or lot-size, which are two important components of a hybrid inventory ordering policy. The results revealed that it is possible to achieve good approximation of energy consumption in the neighborhood of important average point (decision parameter of the inventory model).

[1] The author was enrolled for a DTECH (Mechanical) degree in the Department of mechanical Engineering, University of Vaal, South Africa
*Corresponding author

1. INTRODUCTION AND BACKGROUND

Existing literature shows that operational inefficiency is one important cause of energy consumption problem encountered within most manufacturing plants. Energy consumption can be perceived as an important key performance indicator because it is mostly influenced by the production rate, which in turn is linked to the lot-size and lead-time. Note that both lead-time and lot-size (order quantity) are two important input of every inventory policy [1] [2] [3] [4]. Many inventory-ordering policies have been proposed to address important issues in most manufacturing plants and unfortunately none of them has effectively addressed the existing energy consumption problems. These problems consist of carefully manipulating variable lot-size or orders quantities in order to minimize the inventory cost under a set of appropriate constraints. The present state of inventory costs within most plants is a proof that such issue still needs to be addressed [4]. The concept of inventory control and management is not new in organizations [5] [6] [7] [8]. For instance, the economic order quantity model was developed to help organizations with the problem of determining the optimal quantity to order [9]. In order to improve inventory control in complex and dynamical environment, a probabilistic model was developed [9]. Recently, a hybrid inventory model was developed to enhance inventory control [10]. Although cost-effective, the aforementioned hybrid inventory model presented some drawback, which is an indication for improvement on the modeling approach.

In general, the state of a hybrid inventory model can be described by the values of continuous variables and discrete mode. Information can be seen on references [11] [12], for an introduction of hybrid systems. In the same order of idea, a linear combination technique was used to hybridize both continuous *(r,Q)* and periodic *(R,S)* inventory model. Although the results of implementing such hybrid system were found useful in reducing the total inventory cost, it should be observed that linear equations cannot predict and control effectively the stochastic manufacturing plant performance behavior. At this stage, a new science is therefore needed because the old one is just insufficient. A non-linear combination technique should be used to hybridize both continuous *(r,Q)* and periodic *(R,S)* inventory model. It is then necessary to build a robust hybrid inventory system and test it to see whether it can be used to deal with the issue of efficiency within a manufacturing plant. In this research paper, the proposed hybrid inventory model is seen as the one that both flow and jump. Thus, an appropriate framework meant to study Such hybrid inventory behavior is needed. Note that a hybrid inventory model that flow and jump can be described by a differential equation. An optimization approach that exploits the randomness of the production rate under specific circumstance (down time machine) and studies its effects on the hybrid inventory level is proposed. More attention is finally devoted to formulating accurate model for lot-size with regard to variation in energy consumption, which is perceived as key performance indicator.

One of the things energy consumption change can make happen is the high cost of implementing the proposed hybrid inventory model. Given the fact that the cost of implementing a hybrid inventory model within a manufacturing plant is subjected to variations, energy consumption may be seen as an important factor that prevent such cost to remain as low as possible. This high hybrid inventory cost is mostly due to random production rate. Unfortunately, random production rate mostly impacted by downtime machine is not well monitored. Note that production rate is function of the lead-time and lot-size. Note that both lead-time and lot size are mostly random in most manufacturing plants. Further, lead-time which is an important input of the hybrid inventory model is highly affected by delay (downtime resulting either from material hardness or over production). Here, the lead-time is totally dependent on the lot-size and production rate. Meaning that for a specified lot-size, the lead time will increase if the speed at which items are produced decrease. The aforementioned relationships should be analyzed with enough care because they indirectly lead to energy cost. Hence, it is also required to study the impact of both production rate and lot-size on energy consumption. The analysis of the aforementioned relationship is then justified. Note that a proposed method meant to analyze the relationship between energy consumption and inventory is organized as follows: Theoretical foundation of the model; Properties of both hybrid and energy model; Empirical part of the models (model estimation and empirical study); Simulation study; results.

2. METHODOLOGY

2.1 Case study company background

A case study company is a manufacturing firm around the Moungo region (Cameroun) which produces and sells more than 2 chemical products all over the country. There, products are distributed via company owned distribution. More frequently, the distribution center orders products from the factory, stores them at its facility, and waits for customers (agents or retailer), who then sell the products to their own customers. One of the products that are distributed and that represent the majority of annual sale is analyzed. The current inventory policy being used is periodic review *(R, S)*. At the beginning of each week, the inventory manager of the distribution center reviews weekly sale forecast together with the space available in the distribution center and decides on the quantity to order by using his own experience. Further, the inventory manager orders enough products to prevent stock-out problem. For this study, the company would like to further examine other possibilities such as hybrid inventory review systems that is made of combination of both periodic *(R, S) and* continuous *(r, Q)* review model. In addition, the company also would like to better understand the impact of different factors (random production rate, demand) on energy consumption level. Primary data over a year period were then collected and analyzed in order to fully define and understand the problem (see figure 1).

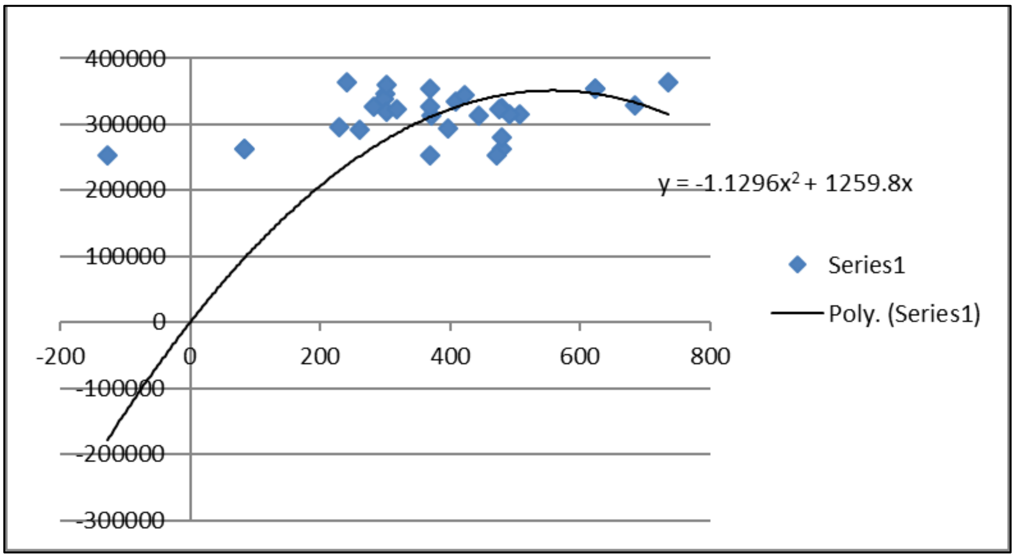

Figure 1: Energy consumption data versus lead-time

As can be seen from figure (1), it is observed a polynomial energy function of degree 2. This is called a quadratic energy function with a lading coefficient of -1.12. It is also observed a concave form (increase and decrease) of energy consumption. Such pattern of energy consumption may come with many disadvantages that are not fully addressed in most research papers. For instance, the polynomial pattern of energy could be most influenced by the random production rate, which in turn is related to lead-time and the lot-size based. Thus, it is then developed an optimal energy inventory model in which energy is derived from energy source. A conceptual approach in which such energy model is derived from past data has been dealt with by many authors [13]. However, most of these models have not been fully developed within a plant, taking into consideration the impact of a implementing a flexible and hybrid inventory ordering policy. This was a good reason to analyze the cost implication of energy consumption under hybrid inventory model that combines the feature of both a periodic and continuous inventory models. The dynamics of such hybrid inventory model is fully described in previous research papers [10]. For a practical demonstration of such hybrid inventory system, read the next paragraphs.

2.2 Hybrid inventory review problem

Genaraly, the annual hybrid cost of ordering, holding inventory and incurring backorder as described by Rossetti [14] [10] [20], provides a good starting point.

$$TC(r,R,Q) = k * N(t)_{hybrid} + h * \frac{1}{T} * \int_0^T I(t)_{hybrid} dt + j * \frac{1}{T} \int_0^T B(t)_{hybrid} dt \qquad (1)$$

Where *TC* is the total cost per unit time of implementing a hybrid *(r, R,Q)* inventory review policy, *r* is the re-order point, *Q* is the lot-size, *R* is the review period, *k* is the order preparation cost per order, *N(t)* is the number of replenishment orders made per unit time, *h* is the holding cost for the item in units per time, *T* is the period or cycle time, $I(t)_{hybrid}$ is the inventory on hand, *j* is the backordering cost for the item in unit per time, and $B(t)_{hybrid}$ is the inventory backordered. It should be noted that the number of replenishment per time $N(t)_{hybrid}$ is dependent on the demand rate and the inventory on hand. $I(t)_{hybrid}$ is dependent on depletion rate.

Number of replenishment $N(t)_{hybrid}$

The features needed to describe Periodic*(R, S)* and Continuous *(r, Q)* review combination system is used to determine the number of replenishment over the period. This hybrid system may depend on the relative numbers of replenishment performed under both review systems. Let $N(t)_{con.h}$ be the number of replenishment performed per period under a Continuous *(r, Q)* review system and $N(t)_{per.h}$ the number of replenishment performed per period under a Periodic *(R, S)* review system needed for hybrid model at a specified capacity utilization level *U*. Represented as a percentage, the capacity utilization is the extent to which the productive capacity of a firm is being used in generation of goods and services. It is then possible to formulate the equation for the total number of replenishment in performing both Periodic *(R, S)* and Continuous *(r, Q)* review system as follows [14] [10].

$$N(t)_{Con_h} * T_{Cont_h} + N(t)_{Per_h} * T_{Per_h} = T * U \qquad (2)$$

$$N(t)_{Con_h} * Q_{Cont_h} + N(t)_{Per_h} * Q_{Per_h} = D_{Total} \qquad (3)$$

$$N(t)_{Cont_h} + N(t)_{Per_h} = N(t)_{Hybrid} \qquad (4)$$

Where *U* is the capacity utilisation during the period in percentage, *T*U* is the total time (number of weeks) of operation per period, $T_{cont.h}$ is the replenishment cycle time under a continuous *(r, Q)* review system in weeks/year, $T_{per.h}$ is the replenishment cycle time under a periodic *(R, S)* review system in week/year, $Q_{con.h}$ is order quantity under a continuous *(r,Q)* review system needed for a hybrid model, $Q_{per.h}$ is the order quantity under a periodic *(R,S)* review system needed for a hybrid model, $N(t)_{hybrid}$ is the total hybrid inventory replenishment.

In this case, the cycle time (time/cycle) under a periodic *(R, S)* inventory system can therefore be expressed by

$$T_{\text{Prev}} = \frac{(Q_p)}{\left(\frac{\partial D}{\partial t}\right)} \qquad (5) \qquad\qquad T_{con} = \frac{(Q_c)}{\left(\frac{\partial D}{\partial t}\right)} \qquad (6)$$

Where Q_p or (Q_{per}) is the order quantity following periodic review system. Further, the cycle time under a continuous *(r, Q)* review system has Q_c or (Q_{cont})*, which* is the order quantity.

Inventory held $I(t)_{hybrid}$

From a mathematical perspective, the total hybrid inventory level may be described by three components: the normal consumption inventory, drop inventory and lead time safety stock inventory. The first one, called “the normal consumption inventory may be described as the inventory that is depleted from the initial stock. A possible model is then.

$$I(t)_{A3} = \int_0^T (I_0 - \mu(t))dt I(t)_{A3} = \int_0^T \left(I_0 - \left(\frac{\partial D}{\partial t} * t \right) \right) dt \tag{7}$$

Where, $I(t)_{A3}$ represents the instantaneous inventory before the re-order (jump process) takes place with, t ($t > 0$) is the time that elapse between the initial time (beginning of operations) to the re-order time, I_0 is the initial inventory, $\mu(t)$ is the degradation/utilization.

The second type of inventory, called "drop inventory" is the one needed to deal with stock-out events when the on-hand-inventory drops below a specified point before the review date R as a result of external factors such marketing and promotion. Note that the quantity needed to cover the expected need (demand) at this particular time can automatically be placed. It implies that there should be restocking times t_r and associated order amounts Q_r. This drop inventory performed through the spontaneous replenishment process may be described by.

$$I(t)_{B3} = \frac{1}{T} \int_0^T (\delta(t - t_r) * Q(t_r)) dt \tag{8}$$

Where δ is the Dirac delta function or step function, t_r is the time at which the jump process is observed and Q_r is the order quantity at time t_r.

The third inventory known as safety stock inventory is the one required for dealing with uncertainty during restocking or replenishment. This safety stock inventory is given by

$$I(t)_{C3} = \left(S_{hybrid} - \left(\frac{\partial D}{\partial t} \right) * T_L(t)_{hybrid} \right) \tag{9}$$

Where S_{hybrid} is the target hybrid inventory level, $T_L(t)_{hybrid}$ is the consumption time or protection demand interval. Hence the instantaneous hybrid inventory held

$$I(t)_3 = I(t)_{A3} + I(t)_{B3} + I(t)_{c3} \tag{10}$$

Inventory backordered $B(t)_{hybrid}$

It should be noticed that the hybrid backorder inventory can be computed by applying the same principle as for the Periodic (R, S) and Continuous (r, Q) inventory review systems. Therefore, the general expression of the instantaneous backorder inventory per time can be given by.

$$B(t)_{Hybrid} = \int_0^T (B_{bo.hybrid} * T_{B.hybrid}) dt \tag{11}$$

Where $$B_{bo.Hybrid} = \frac{1}{period} \int_0^T \left[\frac{N_{bo.hybrid}}{N(t)_{Hybrid}} \right] dt, \tag{12}$$

Where $B_{bo.hybrid}$ is the hybrid backorder rate, $T_{b.hybridt}$ is the hybrid time length of the backorder, $N_{bo.hybrid}$ is the practical number of back order per time and $N(t)_{hybrid}$ is the ideal number of order per period under a hybrid review policy, $T_{bo.hybrid}$ (t) is the time length for the hybrid backorder during a period. Then, $T_{bo.hybrid}$ (t) can also be given by

$$T_{bo.hybrid.}(t) = MLT(t)_{BO\,hybrid} + PROD(t)_{BO_{hybrid}} \tag{13}$$

Where $MLT(t)_{BO.hybrid}$ is lead time for backorder, $PROD(t)_{BO.hybrid}$ is production time for backorder. The value of the decision variables *r, R, S* and *Q* can then be obtained by solving appropriate differential equations.

Energy used is perceived as constraint for most manufacturing plant performances. Since it was demonstrated that the implementation of a hybrid inventory model can bring more value in most systems, the question now is to understand its implication on other performances. Note that energy consumption's behaviour and structure within most manufacturing plant can be analysed differently. In this case, one had to figure out what the true dynamic of energy is with respect to inventory. How to put it in way that most managers could understand what the true dynamic of energy is? So, part of the development undertaken in the next sections is dealt with that question.

2.3 Properties of the energy model

It is almost impossible to determine the energy consumption pattern of electric equipment without a good understanding of the power theory. So, the term power theory of electrical installation can be understood as the state of knowledge on their power properties. In that sense, it is a set of true statements, interpretations, definitions and equations describing these properties [15]. Such equation of power in given as follows [16] [17]:

$$P = \frac{1}{T}\int_0^T p(t)dt = V_{RMS} * I_{RMS} \therefore P = S\cos\varphi \tag{14}$$

Where V_{RMS}, I_{RMS} are the effective value of the varying voltage and current, then *S* is the apparent power, $\cos\varphi$ is the phase difference between the current and voltage drop across the load.

2.4 Empirical part of the model (model estimation and empirical study)

For instance, energy consumption that results from implementing a hybrid inventory model within a manufacturing plant hang in the balance, it is uncertain. This may be due to the random nature of production rate, which in turn is linked to the lot-size and lead-time. Hence it is important for production rate to remain consistent with the inventory model's parameters, and it can be done by carefully handling the random behavior of lot-size and lead-time. In this research paper, mass balance (inventory level)for unsteady state process is dealt with in order to lower or keep energy consumption as much steady as possible. So, in term of interpretation, this energy consumption is then developed as follows:

$$E(t) = \sum P * M_{LT} = \sum_{i=1}^{\infty} P * \frac{1}{Th_r} * Q(t) \tag{15}$$

Where Th_r is the theoretical processing rate (speed for actual product output), *P* is the electrical power consumed by the equipment of the production line, $M_{Lt}(t)$ is the manufacturing lead time, and *Q* is the lot-size of implementing a hybrid inventory model. Equation (15) was inserted into the energy balance equation that result from producing item, which is meant to represent what could had happened to the plant by looking at the difference between the new and old value of energy. A possible model form of energy balance in this case is then given by

$$\frac{dE(t)}{dt}=\frac{d\left(P*\frac{1}{Th_r}*Q(t)\right)}{dt}=P*\frac{1}{Th_r}*\frac{dQ(t)}{dt} \tag{16}$$

Note that equation (16) is seen as a non-linear differential equation. From the aforementioned equation, the basic idea is to make a change of variables in order to come up with a different form of the lot-size by reshaping it. In general, the temporal variation of energy consumption is defined by

$$\frac{dE(t)}{dt}=\frac{E(t_2)-E(t_1)}{t_2-t_1} \tag{17}$$

Where *dE/dt* is the variation in energy consumption, E_2 is the energy consumed during the second period t_2, E_1 is the energy consumed during the first period t_1. Note that t_2 and t_1 describes the fluctuating lead time. Equation (16) is equated to equation (17) in order to have a clear expression of the lot-size (Order quantity *Q*) pattern. This led to the following lot-size balance for the lot-size in the plant:

$$P*\frac{1}{Th_r}*\frac{dQ(t)}{dt}=\frac{Th_r}{P}*\left[\frac{E(t_2-E(t_1))}{t_2-t_1}\right] \tag{18}$$

Observe that the proposed hybrid inventory model is governed by a continuous and discrete dynamics. In other word, the hybridization takes place because of the dynamical switching process from continuous to discrete. Of the many basic modeling approaches for hybrid system, an explicit approach is used to describe and control the evolution of the lot-size *Q*. This explicit approach is described by cautious equations that lead to a system of differential equations.

$$\frac{dQ(t)}{dt}=\rho*E(t_2)-\rho*E(t_1) \tag{19}$$

The lot-size as described by equation (19) is subject to change that depends only on initial state (before) and final state of the energy consumption, but not on the manner used to realize the change in energy consumption. One could then think about the manner used to realize such change from the initial to the final state of energy consumption. Equation (6) becomes:

$$E(t_2)-E(t_1)=\frac{1}{\rho}*\frac{dQ}{dt}*(t_2-t_1) \tag{20}$$

Where E(t_2) is the new value of energy consumption under a continuous inventory model, $E(t_1)$ is the old value of energy consumption under the hybrid inventory model, φ is the production rate, t_2-t_1 is the change in time. It may be that there is evidence that the instantaneous rate of change at time t is actually a function of previous time. This is referred to as a delay differential equation. Equation (20) logically leads to a continuous change in energy.

$$\frac{dE}{dt} = \frac{1}{Th_r} * dQ * (E_2 - E_1) = \alpha * E \tag{21}$$

Equation (21) is the energy balance equation and represents the increase in total energy as result of change in time by one unit. It shows that the new value of energy consumption is dependent of the old value of energy consumption and the effect of variation of the lot-size. In other word, energy balance is the relationship between new value of energy consumption and old value of energy consumed. In this case, the difference between *E*(t_2) *and E*(t_1) shows up as the change in energy consumed by the plant.

2.5 Solving the mathematical problem

Equation (21) can be rearranged by approximation. Of course, additional terms can be added that modify the evolution of the change in energy consumption. A possible approximation of such change in energy consumption over time is obtained using the Taylor series method as follows.

$$E(t) = \alpha_1 * E_T{}^1 + \alpha_2 * E_T{}^2 + \alpha_3 * E_T{}^3 + ...\alpha_n * E_T{}^n \tag{22}$$

The parameter α meant for the flux rate can then be set as follows:

$$\alpha = C^{te} \therefore \frac{dE_T}{dt} = \left[\alpha_1 * E_T{}^0 + 2 * \alpha_2 * E_T{}^1 + 3 * \alpha_3 * E_T{}^2 + ...\right] * \frac{\partial E_T}{\partial t} \tag{23}$$

$$\alpha \neq C^{te}$$

$$\frac{dE_T}{dt} = \left[\frac{d\alpha_1}{dt} * E_T{}^1 + \alpha_1 * \frac{dE_T{}^1}{dt}\right] + \left[\frac{d\alpha_2}{dt} * E_T{}^2 + 2 * \alpha_2 * E_T * \frac{dE_T}{dt}\right] + \left[\frac{d\alpha_3}{dt} * E_T{}^3 + 3 * \alpha_3 * E_T{}^2 * \frac{dE_T}{dt}\right] + ... \tag{24}$$

Important input data and parameters of such hybrid inventory model were collected from archival record. important parameters needed for this study are represented as follow: The ordering cost *k=R125 per order*; The holding cost *h=R0.511 per unit per time*; The back order cost *j=R201per unit per time*; The service level was set to 90%

3. SIMULATION OUTPUT RESULTS

In this section, a simulation/experiment was performed on the constructed mathematical models. Note that Engineering Equation Solver (EES) is the software used to run the energy models. The simulation output results are consolidated in data reports and presented by graphs. The graphs obtained from simulation studies are useful in that they can raise questions, which in turn stimulate further investigation.

3.1 First output result

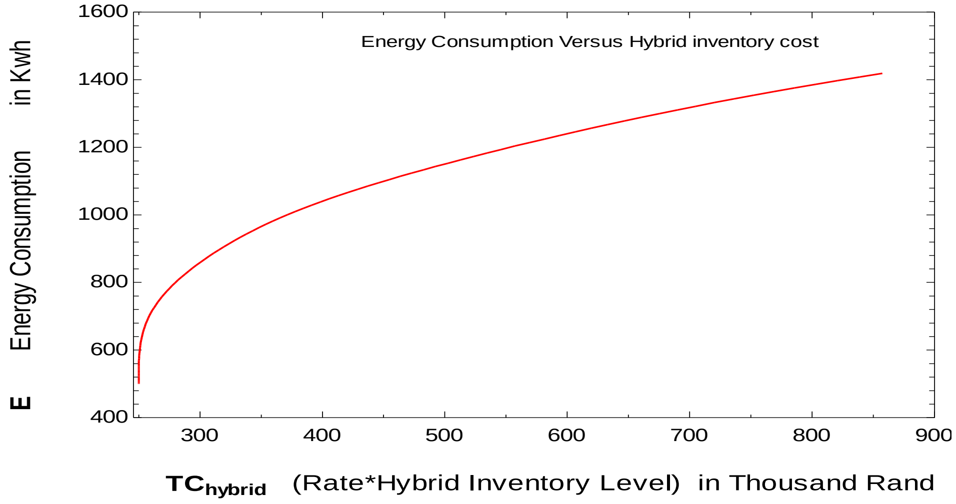

Figure 2: Energy consumption versus hybrid inventory cost

It is noticed an unusual correlation between energy consumption and hybrid inventory cost. Pointing this fact out under this section is perceived as the starting point of an effective analysis. It is observed a particular path of energy consumption, which of course does not, tends to infinity really fast but rather is subject to stabilization. One shouldn't worry about such type of energy behavior for at least many reasons. Notice on the graph 1 that as the hybrid inventory cost increases, the energy consumption also increases. It is clear that such energy consumption function is an increasing function. Since an increasing function is observed (a one to one relationship), it is destined to have a unique inverse. It is also observed on figure (1) that an increase in the value of energy consumption is really considerable between two point of the inventory cost (300 and 400 thousand rand). After the hybrid inventory cost that equate 400 thousand rand, as the hybrid inventory cost gets larger and larger, the increasing evolution of energy consumption start slowing down. The aforementioned relationship between energy consumption and hybrid inventory happened for an important reason. The presence of the logarithmic growth of energy consumption may come from the fact that the production rate increases in a -fixed percentage. Such event must compels one to understand what could happen if the production rate does not increase at a fixed percentage. The product that is manufactured or processed is made up of some type of material, which of course may impact the speed. A reasonable answer could be obtained from signs. For instance, a visible sign of plant ineffectiveness may be perceived as the impact of production rate change. Hence it is crucial to handle the speed at which the products are produced.

A logarithmic growth of energy consumption like that shown in figure (1) is often not well understood by the decision makers. In two important points (300 to 400), the amount of energy consumed has significantly increased. The logical consequence of such energy consumption pattern is that it may get out of hand over time and affect the hybrid inventory cost. Such logarithmic growth of energy consumption is then justified by the fact that demand and lead-time fluctuation have significant implication on production rate, which in turn is linked to energy consumption. In this particular case, the lead-time may be mostly impacted by the downtime machine. Problems like that become easy to be recognized, because of the large number of activities in the production process [18]. For example extended lead-time (made up of value and non-value activities) can lead to poor throughput rate, which in turn has a direct impact on the economic performance/effectiveness. Mostly, the value activities are lot-size related. Knowing the link that exists between lot-size and lead-time, it is crucial to focus more on value activities. This is a clear indication that a proper management of production rate through the lot-size and lead-time management is capital. Otherwise, it may have negative implications on energy consumption in the long run (see figure 3). Although this research paper is able to provide some insights, it is acknowledged a limitation. For instance, it is used various adjustments of the model parameters and forcing within the range of the uncertainties without emphasizing on the right number of replications. However, much could have been done if an appropriate number of replications were done. Future research should include model calibration with multiple and appropriate replications.

3.2 Second output result

In this section, it is used a 3Dimensional graph scatter plot to see how the production rate relate to the hybrid inventory policy that is implemented and the energy consumption. This three Dimensional graph is used to explore three important aspects: the strength; direction; and non-linearity of the relationship between the three aforementioned variables.

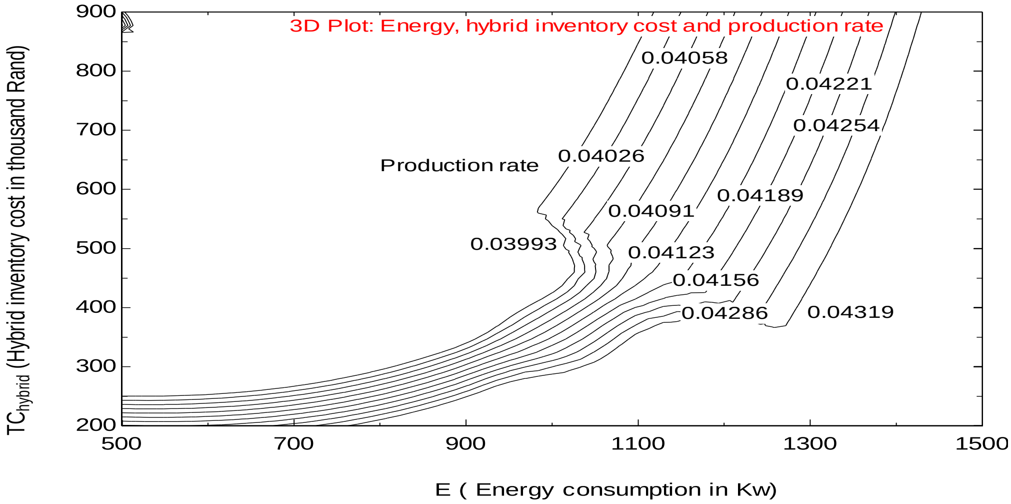

Figure 3: Impact of production rate on energy consumption and hybrid inventory cost

As can be seen from figure (2) it is observed a system with a production that seem to keep a constant path until an unexpected even happen. Most goods have mechanical properties that must be taken into consideration during the manufacturing process. The amount of time required to produce one unit of good may then be set accordingly because it can reflect on the system flow rate. The same level of activity indicates that such system is moving on an easier pace (move at a particular pace). That means, it is working much efficiently. However, as the production rate changes suddenly, it might seriously impact on the energy consumption and hybrid inventory cost. Studies for the use of energy in production showed that different production rate can generate different levels of energy consumption [19] [20]. Manufacturing firms have recognized this trend and are now trying to implement corporate strategies so as to improve energy efficiency [19].

Recall that figure (2) portrayed a hybrid inventory cost that tend to follow an unusual path with respect to energy consumption, but is permutated by the change in the production rate. Further, figure (2) showed that the pattern of energy consumption versus a hybrid inventory model is totally affected by the speed at which the items are produced. The aforementioned path is unstable as result over time. Launching a production with low speed resulted in reduced energy consumed while the hybrid inventory cost increased. However, having a production line that performs at a high speed compelled the production line to have high energy consumption while the hybrid inventory cost was low. Future research should then emphasize on the balance between energy consumption and production output.

3.3 Third output result

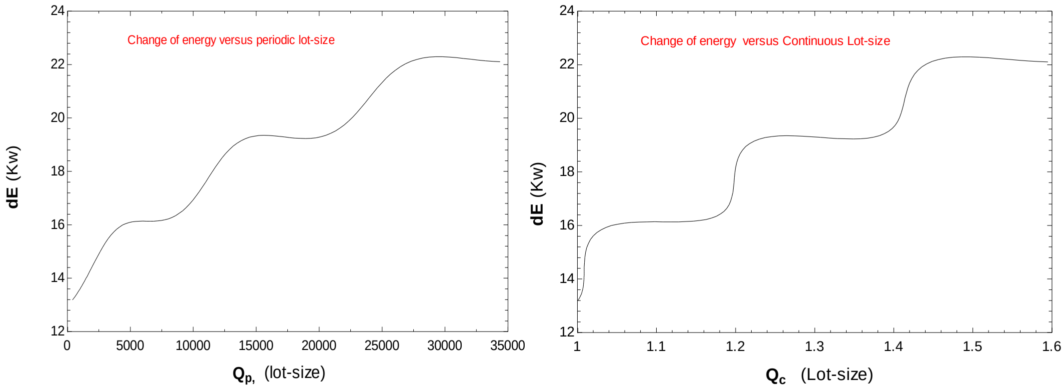

Figure 4: impact of the lot-size on the change of energy

It is presented a graph describing how the level of change of energy had increased with respect to the lot size. The step function in this case is found useful in representing the data as the nature of the change of energy is not naturally continuous. The energy function is increasing on the interval (0,500), constant on the interval (800,1000) and increasing again on the interval (1000, 1100). This function appeared to be a step function, which is one kind of piecewise function defined by constant value over each part. Note that a conclusion that is drawn from the graph portrays that step graph can describe both change of energy and discrete nature of the change due to sometime the constant movement of the lot size.

4. MODELS VERIFICATION AND VALIDITY

The independent models are run such that artificial data set (figure 2) be generated and be checked against the real data that were seen on figure (1). In other word, it is checked whether the same pattern of energy consumption versus time emerged after running the models. As can be seen from real data (figure 1) and simulation output result (figure 2), the same pattern of energy consumption versus Hybrid inventory cost almost emerged. Note that such hybrid inventory cost model is function of time. In this case, there is a relation between energy consumption and hybrid inventory cost, which in turn is related to time. Thus, energy consumption is related to time. Such equality is a transitive relation, which of course compelled one to say with a certain degree of confidence that the hypothesis/theory was validated. It should be further noticed that some results that confirm the validity and verification of the proposed models are displayed on figure 2 and 3.

5. CONCLUSION

Energy consumption is an interesting property of most manufacturing plants and items produced might affect the occurrence of energy consumption. Managers should be aware of the risk factors for higher energy consumption, which may be proven useful in identifying items produced at high risk for energy consumption peak. This can help one to minimize the risk of higher energy consumption through the avoidance of all the unnecessary activities that may cause different type of breakdown machine (or stoppage), which in turn are tied to the production rate. In other word, everything has to be done at an easy pace. That means it is happening efficiently. A system with the right vision will have higher efficiency than the others.

6. REFERENCES

[1] Kotler, P. 2002, Marketing Management: The Millenium Edition*,* 2nd Ed, New Delhi: Prendice Hill Of India

[2] Kumar. J & Pradeep, K.S. 2013. Survey Of Demand Fluctuation Of Dairy Products At Raipur Dugdh Sangh, Chhattisgarh, India For Selection Of Appropriate Forecasting, *Uncertain Supply Chain Management*, 1 (2013) 133–144

[3] Chalotra, V, 2013, Inventory Management And Small Firms Growth: An Analytical Study In Supply Chain. Vision, *Sage Journal*, 17(3), 213-222
[4] Isaksson, O.H.D. & Seifert, R.W, 2014, Inventory Leannes And Financial Performance Of Firms. Production Planning & Control: *The Management Of Operations Journal*, 25(12), 999-1014
[5] Anichebe, N.A & Agu,O.A. (2013), Effect Of Inventory Management On Organizational Effectiveness, *Information And Knowledge Management*, 3 (8), 92-100
[6] Mohamad, S.J & Suraidi, N.N & Abd.S.N & Suhaimi,R..D. 2016, A Study On Relationship Between Inventory Management And Company Performance: A Case Study Of Textile Chain Store, *Journal Of Advanced Management Science*, 4(4), 299-204
[7] Mpawanya, F, 2005, Inventory Management As A Determinant For Improvement Of Customer Service*, Master Thesis, Pp 23-24*
[8] Sietienei, E & Memba, F. 2015, The Effect Of Inventory On Profitability Of Cement Manufacturing Companies In Kenya: A Case Study Of Listed Cement Manufacturing Companies In Kenya, *International Journal Of Management And Commerce Innovations*, 3(2), 111-119
[9] Harris, F. 1913. How Many Parts To Make At Once, *Factory, The Magazine Of Management*, 38(6), 947–950.
[10] Benga,E.B.B. 2015. Optimizing The Inventory Ordering Policies In Supply Chain Management, *Thesis Submitted In Fulfilment Of The Requirements For The Degree Magister Technology In Industrial Engineering . 52-54-16*
[11] Lunze,J & Lammabhi-Lagrigue, F. 2012. Even Based Control With Communication, *22 Osilar Conference On Signal & System And Computers*
[12] Perona, M & Saccani, N & Zanoni, S . 2009. Combining Make-To-Order And Make-To-Stock Inventory Policies, *An Empirical Application To A Manufacturing Sme Marco*
[13] Neshat, N & Amin-Naseri, M.R. 2014, "Energy Models: Methods And Characteristics" Journal Of Energy In Southerrn Africa, 25(4), 101-110
[14] Momeni, Z & Azizi, A. 2018. Current Order And Inventory Models In Manufacturing Environments: Areview From 2008 T0 2018*, International Journal Of Mechanical And Production Engineering Research And Development*, 8(1) 1087-1096
[15] Seitbmetz, C.P. 1997. Theory Of Alternating Current Phenomena, New York
[16] Czamecki, J.S.1997. Power And Compensation In Circuits With Periodic Voltage And Currents, *Outline Of The History Of Power Theory Development*, 3(2), Pp 37-46
[17] Schulze, S & Seuring,S Ewering,C, 2012. Applying Activity-Based Costing In A Supply Chain Environment, *Int. J. Production Economics*, 135(1), Pp. 716-725,
[18] Kumar,R. & Sundar, R & Balaji, A, 2014. A Review On Lean Manufacturing Implementation Techniques, *Procedia Engineering*, 97(1), Pp. 1875-1885.
[19] Fysikopoulos, A & Alexios, P & Georgios, S & George, C, 2013. Energy Efficiency Of Manufacturing Processes: A Critical Review, *46 CIRP Conference on Manufacturing Systems.*
[20] ROSSETTI, M.D. 2010. Simulation Modeling & Arena. 1st ed. Hoboken: John Wiley & Son,Inc
[21] Liang, Y. 1997. The Development Of An Intelligent Inventory System, *Thesis, Usur*