



A ROBUST MATHEMATICAL MODEL FOR ROUTE PLANNING OF A THIRD-PARTY LOGISTICS PROVIDER

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ABSTRACT

Third-party logistics (3PL) is gaining ground as companies try to focus on their core competencies and outsource their logistics activities. However, the environment is becoming more and more competitive for 3PL providers as new rivals enter the market. Routing, as one of the main activities of 3PL, has a major share in transportation costs and apparently should be planned cautiously. In addition, route planning in real-world situations is subject to uncertainties which may greatly affect the performance of a 3PL provider if neglected. Having these concepts in mind, a robust mathematical model for route planning problem in a 3PL company is proposed. The problem is modeled as a robust multi-depot multiple travelling salesman problem (MmTSP) and numerical results are provided and a series of sensitivity analyses are conducted to show that the proposed model is promising.

Keywords: Robust Optimization, Route Planning, Third-party Logistics Provider, Uncertainty

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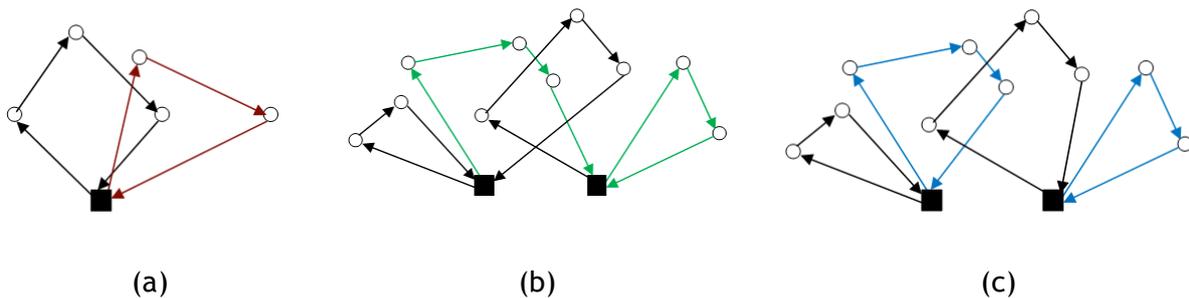
1 INTRODUCTION

The increasingly competitive business environment forces the companies to concentrate on their core competencies and outsource their non-primary activities to firms that are specialized in such operations and/or services. More specifically, the great share of logistics in costs incurred by the companies prepared the bed for the emergence of the third-party logistics providers. A third-party logistics provider (3PL) can be defined as an outsourced provider that is responsible for managing all or part of its customer's logistics activities including warehousing, transportation, distribution, cross-docking, packaging and freight management. However, the definition may vary from contract to contract. From another point of view, the network of a 3PL is basically different from that which is owned by a company. The main objective of the supply chain own by a company is to manage the operations within the same organization while a 3PL has to deal with a number of various clients. The logistics market has had a significant growth in the last decade and, in turn, the environment for the third-party logistics providers has been increasingly competitive. As an example, according to a report by Armstrong & Associates, Inc., the total revenues of 3PL market size in the U.S. rose from \$89.4 in 2004 to \$127.3 in 2010 (Armstrong & Associates 2012).

The growth in the applications of 3PL in the real-world situations encourages the researchers to investigate different aspects of the problem. Especially in designing a 3PL network, one may mentioned the paper by Ko et al. in which a hybrid optimization/simulation approach for distribution network design for 3PLs is proposed (Ko, Ko and Kim 2006). In another work, a genetic algorithm-based heuristic is proposed by Ko and Evans for the integrated dynamic integrated forward/reverse logistics network for 3PLs (Ko and Evans 2007). Jung et al. proposed a decentralized supply chain planning framework based on minimal-information sharing between the manufacturer and the 3PL provider (Jung, Chen and Jeong 2008). For a survey on the concepts of 3PL one may refer to a review paper by Marasco (Marasco 2008). A distribution network optimization problem is solved in a paper by Başlıgil et al. following a two-stage solution strategy in which the first stage is solved by GAMS/CPLEX and the results of the first stage is used in the second stage where genetic algorithm is applied (Başlıgil, et al. 2011).

Transportation as one of the major activities of any asset-based third-party logistics provider may be divided into sub-tasks. One of the most important issues in transportation is route planning which may impose great costs to the firm if neglected. Vehicle Routing Problem (VRP) is one of well-known problems formulated to model the routing problem in real-life applications. VRP and many other problems can be reduced to the important well-studied model called the Travelling Salesman Problem (TSP) or its variants (Bektas 2006). Assume that a salesman starts from his home city and visits each city on his tour exactly once and then returns home; the TSP is to find the tour with minimum distance traveled by the salesman. Considering multiple travelling salesmen in the TSP results in a problem called the standard Multiple Travelling Salesman Problem (mTSP) which is to find the best tours for the m salesmen who all depart from and return to a single node known as the depot. A sample mTSP is depicted in Figure 1 (a) where the number of cities is 5 ($n = 5$) and the number of salesmen is 2 ($m = 2$). In a paper by Bektas, the literature on mTSP is investigated from an application point of view and integer programming formulations of the problem and their specific solution methods are provided (Bektas 2006). In the Multi-depot mTSP (MmTSP), a generalization of the mTSP, multiple depots are considered. In each depot in MmTSP, there are a predefined number of salesmen and the tours should be find such that all customers are visited exactly once and the number of customers visited by each salesman lies between a predetermined lower and upper bounds and the total cost of all the tours are minimized. Assuming that all the salesmen should return to the depot to which they belong, leads to a version of the MmTSP called the *fixed destination MmTSP* and relaxing such constraint results in another version called *non-fixed destination MmTSP*. However, in the non-fixed

destination MmTSP, the number of the salesmen in each depot should remain the same when the salesmen finish their tours (Kara and Bektas 2006). Figure 1 (b) illustrates a sample non-fixed destination MmTSP where there are 5 cities ($n = 5$) and 2 depots ($d = 2$) each of which has 2 salesmen ($m_1 = 2$ and $m_2 = 2$). Figure 1 (c) shows a fixed destination MmTSP with the same parameters as in Figure 1 (b). Chan and Baker expanded the MmTSP to include vehicle range and service frequency requirements and proposed a heuristic solution method for the problem (Chan and Baker 2005). In a paper by Malik et al. an algorithm with an approximation factor of 2 for a generalized MmTSP when the costs are symmetric and satisfy the triangle inequality is proposed (Malik, Rathinam and Darbh 2007). In this paper, we used the non-fixed MmTSP formulation to model the routing in a 3PL provider in order to find the optimal decision regarding the route planning problem in such firms.



**Figure 1. (a) mTSP ($n=5, m=2$),
(b) Non-Fixed Destination MmTSP ($n=10, m=4, d=2, m_1=2, m_2=2$) and
(c) Fixed Destination MmTSP ($n=10, m=4, d=2, m_1=2, m_2=2$)**

From another perspective, the uncertainty and fluctuations of the market force decision makers to take more reliable and robust measures while each decision level (strategic, tactical and operational) is affected in some way by the uncertainty of the environment. Transportation and route planning is not an exception in this regard and is also affected by uncertainty. To be more specific, the uncertainty in costs associated with moving from one node to another in the supply network may be caused by traffic congestion, weather conditions, fuel price fluctuations, etc. One of the most popular tools for modeling uncertainty is stochastic programming (Gupta and Maranas 2003, Hingle and Wallace 2003). Especially in network design stochastic programming is a favorite tool for researchers (Santoso, et al. 2005, Lieckens and Vandaele 2007, Lecluyse, Woensel and Peremans 2009, Chen, et al. 2010, Döyen, Aras and Barbarosoğlu 2011). The robust optimization methods belong to a special category of stochastic models in which the expected cost objective is replaced by one capturing cost variability and feasibility (Takriti and Ahmed 2004). Specifically, Sungur, Ordóñez and Dessouky investigated the VRP formulation proposed by Miller-Tucker-Zelmin and specific uncertainty sets to show that solving a robust VRP (RVRP) considering demand uncertainty is no more difficult than solving a single deterministic VRP (Sungur, Ordóñez, & Dessouky, 2008). Mulvey et al. proposed a framework for robust optimization including two types of robustness: “solution robustness” meaning that the solution is nearly optimal in all scenarios and “model robustness” meaning that the solution is nearly feasible in all scenarios (Mulvey, Vanderbei and Zenios 1995). In other words, solution robustness can be defined as proximity of solution to optimality for any scenario and model robustness may be defined as the proximity of a model to feasibility (Leung and Wu 2004). In this paper, we follow the approach proposed by Mulvey et al. and described by Feng and Rakesh (Feng and Rakesh 2010) to model the uncertainty in the route planning in a 3PL provider.

The exposition of this paper is as follows: in Section 2, the problem is formulated and described as a robust MmTSP. Numerical test results are provided in Section 3 to justify the model and Section 4 concludes the paper.

2 PROBLEM DESCRIPTION

We consider a 3PL provider owning a number of depots each of which is equipped with a predetermined number of vehicles. The problem is to find the optimal routes for the vehicles such that all the customers are served by exactly one vehicle considering the uncertainty in the cost associated with the movement of vehicles between nodes of the supply network. This problem is described in details in what follows.

2.1 Assumptions

Proper assumptions about the problem simplify modeling process while keep a satisfying level of reality. Having this fact in mind, the problem is investigated under the following assumptions.

- 1- The supply network of the 3PL provider is presented by a graph consisting of nodes (vertices) and edges (arcs).
- 2- The nodes in the network are of two types: depots and customers.
- 3- The number and location of the depots and customer nodes are known and fixed.
- 4- The number of vehicles (salesmen) in each depot is known and fixed.
- 5- The problem is subject to uncertainty of the cost associated with moving from a node to another and other parameters are deterministic.
- 6- The uncertainty is modeled as scenarios following Mulvey et al. (Mulvey, Vanderbei and Zenios 1995).
- 7- The cost associated with moving from a node to another is known under each scenario.
- 8- Each customer is served exactly by one vehicle.
- 9- The number of vehicles in each depot remains the same before and after all customers are served. However, the depots may exchange vehicles.

2.2 Mathematical Formulation

The graph for the supply network of the 3PL provider is denoted by $G = (V, E)$ where $V = V' \cup D$ is an ordered set of n vertices (nodes) and E is the set of edges (arcs). The first d nodes in V are the depots each of which has a certain number of salesmen. More specifically, in each depot i in $D = \{1, 2, \dots, d\}$ there are m_i salesmen and the total number of salesmen is m . The nodes that are not in D are the nodes which represent the customers; i.e. the set of customers is $V' = \{d+1, \dots, n\}$. Let x_{ij} be a binary variable and equal to 1 if the edge (i, j) is in the optimal solution and 0 otherwise. In addition, suppose that u_i is the number of nodes visited by any traveler on his path from the depot up to a node i . It is assumed that the number of nodes visited by any traveler lies between a lower bound, K , and an upper bound, L . Following (Mulvey, Vanderbei and Zenios 1995), we use ψ_ξ to represent the cost function for each scenario $\xi \in \Omega$ where Ω is a finite set of scenarios each of which may realize with a probability of P_ξ . In this paper we assume that the uncertainty only exists in the cost function parameters and those of constraints are deterministic and therefore, the infeasibility of the model under any scenario will be equal to 0, i.e. $\delta_\xi = 0$. Hence, if λ denotes the weight placed on solution variance, following the

linearization approach chosen by (Mirzapour Al-e-hashem, Malekly and Aryanezhad 2011), the robust non-fixed MmTSP model can be formulated as follows.

$$\min z = \sum_{\xi \in \Omega} p_{\xi} \psi_{\xi} + \lambda \sum_{\xi \in \Omega} p_{\xi} \left[\left(\psi_{\xi} - \sum_{\xi' \in \Omega} p_{\xi'} \psi_{\xi'} \right) + 2\theta_{\xi} \right] \quad (1)$$

Subject to

$$\psi_{\xi} = \sum_{(i,j) \in E} c_{ij}^{\xi} x_{ij}, \quad \forall \xi \in \Omega \quad (2)$$

$$\psi_{\xi} - \sum_{\xi \in \Omega} p_{\xi} \psi_{\xi} + \theta_{\xi} \geq 0, \quad \forall \xi \in \Omega \quad (3)$$

$$\sum_{j \in V'} x_{ij} = m_i, \quad \forall i \in D \quad (4)$$

$$\sum_{i \in V'} x_{ij} = m_j, \quad \forall j \in D \quad (5)$$

$$\sum_{i \in V'} x_{ij} = 1, \quad \forall j \in V' \quad (6)$$

$$\sum_{j \in V'} x_{ij} = 1, \quad \forall i \in V' \quad (7)$$

$$u_i + (L-2) \sum_{k \in D} x_{ki} - \sum_{k \in D} x_{ik} \leq L-1, \quad \forall i \in V' \quad (8)$$

$$u_i + \sum_{k \in D} x_{ki} + (2-K) \sum_{k \in D} x_{ik} \geq 2, \quad \forall i \in V' \quad (9)$$

$$x_{ik} + x_{ki} \leq 1, \quad \forall k \in D, \forall i \in V' \quad (10)$$

$$u_i - u_j + Lx_{ij} + (L-2)x_{ji} \leq L-1, \quad \forall i, j \in V', i \neq j \quad (11)$$

$$\theta_{\xi} \geq 0, \quad \forall \xi \in \Omega \quad (12)$$

$$x_{ij} \in \{0,1\}, \quad \forall i, j \in V \quad (13)$$

where equation (1) represent the objective function under uncertainty. Together with (2), (3) and (12), equation (1) provides the robustness of the model. Constraints (4) and (5) guarantee the proper outbound and inbound movements of salesmen from and to the depots, respectively. More specifically, the number of salesmen (vehicles) in each depot i is m_i and it should remain so before and after all the customers are served. Equations (6) and (7) model the fact that a salesman should leave the customer after the visit and either head

for another customer or for a depot. Inequalities (8) and (9) confine the number of customers a salesman visits to lie between a lower bound, K , and an upper bound, L . Obviously, a salesman may not serve only a single customer; this fact is modeled by (10). Inequality (12) is known as Sub-tour Elimination Constraint (SEC) which breaks all sub-tours between customer nodes. Finally, constraints (12) and (13) determine the types of the variables in the model.

Table 1. Transportation Costs Under Different Scenarios

Scenarios	Origins	Destinations					
		AT	CH	CI	HO	LA	MO
High	AT	0	712	464	852	2496	1296
	CH	712	0	334	1193	2236	784
	CI	464	334	0	1237	2280	898
	HO	852	1193	1237	0	1716	1957
	LA	2496	2236	2280	1716	0	2990
	MO	1296	784	898	1957	2990	0
Moderate	AT	0	705	457	845	2399	1199
	CH	705	0	328	1097	2139	768
	CI	457	328	0	1139	2184	799
	HO	845	1097	1139	0	1619	1859
	LA	2399	2139	2184	1619	0	2905
	MO	1199	768	799	1859	2905	0
Low	AT	0	702	454	842	2396	1196
	CH	702	0	324	1093	2136	764
	CI	454	324	0	1137	2180	798
	HO	842	1093	1137	0	1616	1857
	LA	2396	2136	2180	1616	0	2900
	MO	1196	764	798	1857	2900	0

3 NUMERICAL RESULTS

A test problem is solved to verify the model and a series of sensitivity analyses are conducted to show the effects of different scenarios and parameter changes on the obtained robust optimal solution. In these experiments a 3PL is modeled as a non-fixed MmTSP with $n = 4$, $m = 3$, $d = 2$, $m_1 = 1$ and $m_2 = 2$. The supply network has 6 nodes and is consisted of

two depots that are assumed to be Atlanta and Chicago and the four other nodes which are Cincinnati, Houston, Los Angeles and Montreal. The tour lengths are assumed to be bounded by $K = 2$ and $L = 5$. Three scenarios are considered under which the costs of transporting the goods between the nodes may be *high*, *moderate* and *low*. Table 1 shows the costs between each pair of nodes under the aforementioned three scenarios. It is assumed that the transportation cost is proportionate to distance between nodes in the network and therefore the distances are used as costs.

For the first experiment, the model is solved assuming that the moderate scenario will occur with probability of 1 leaving 0 probabilities for the other two ones. Assuming such probabilities for the scenarios, leads to a basic model ignoring the uncertain aspects of the problem. Table 2 shows the optimal solution and its total cost for this experiment. In this table, V_{ij} represents vehicle j depart from depot i . Since the problem is modeled as a non-fixed MmTSP, in the optimal solution, the depots exchange vehicles; i.e. V_{11} starts its tour from depot 1 and ends up in depot 2 and V_{22} moves from depot 2 to depot 1. However, the number of vehicles in each depot remains the same when the vehicles visit all customers and return to depots.

As our second experiment the *high*, *moderate* and *low* scenarios are given non-zero probabilities equal to 0.1, 0.6 and 0.3, respectively. In addition, the weight for the cost of variance in solutions is assumed to be $\lambda = 3$. The effects of uncertainty can be seen in this experiment considering full features of the proposed robust model. Similar to the previous experiment, Table 3 presents the optimal solution for this experiment. Due to the uncertainty in the problem, the obtained solution is obviously different from that of the first experiment. The total cost in this experiment is consisted of two main parts including the routing costs and the cost of solution variance.

Table 2. The Optimal Solution For The First Experiment

Vehicle	Route	Cost
V_{11}	AT → CI → CH	785
V_{21}	CH → HO → LA → CH	4855
V_{22}	CH → MO → AT	1967
Variance Cost		0
Total Cost		7607

To investigate the effects of the parameter, λ , on the routing cost, solution variance cost and the total cost of the systems, the model is solved considering different values for λ . Figure 2 illustrates the effect of changes in this parameter on the routing cost. Obviously, the increase in the penalty for the variation in solution leads to an increase in the routing cost. On the other hand, as λ increases, the solution variance penalty cost decreases (see Figure 3). It can be concluded that there is a trade-off between these two costs. However, increasing the penalty parameter, λ , causes an increase in the total system cost. Hence, the value of the parameter should be selected based on the importance of the solution variance in each problem.

Table 3. The Optimal Solution For The Second Experiment

Vehicle	Route	Cost
V_{11}	AT → MO → CH	1976.2
V_{21}	CH → CI → AT	784.2
V_{22}	CH → LA → HO → CH	4881
Variance Cost		228.96
Total Cost		7870.36

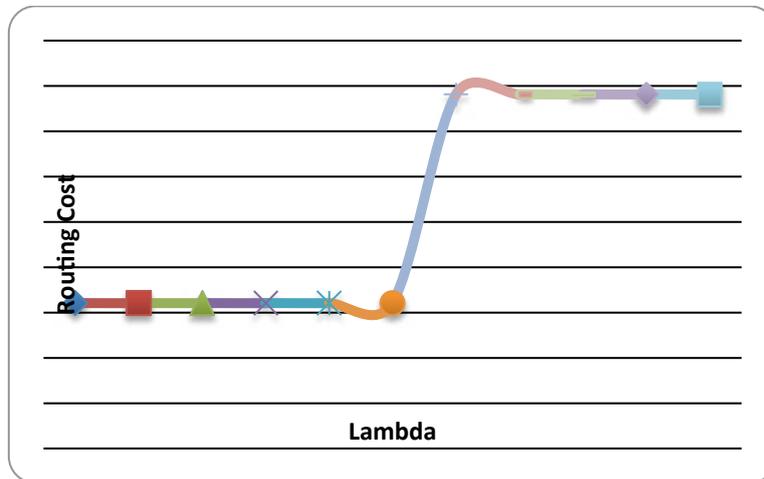


Figure 2. The Effect Of Changes In λ On The Routing Cost

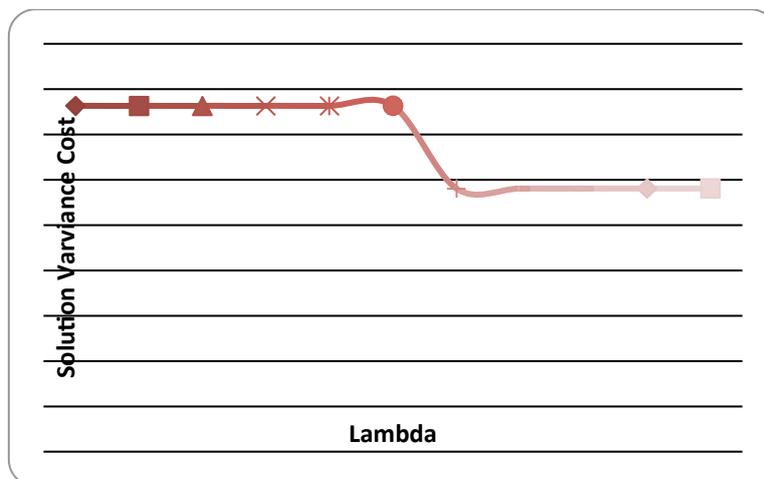


Figure 3. The effect of changes in λ on the solution variance cost

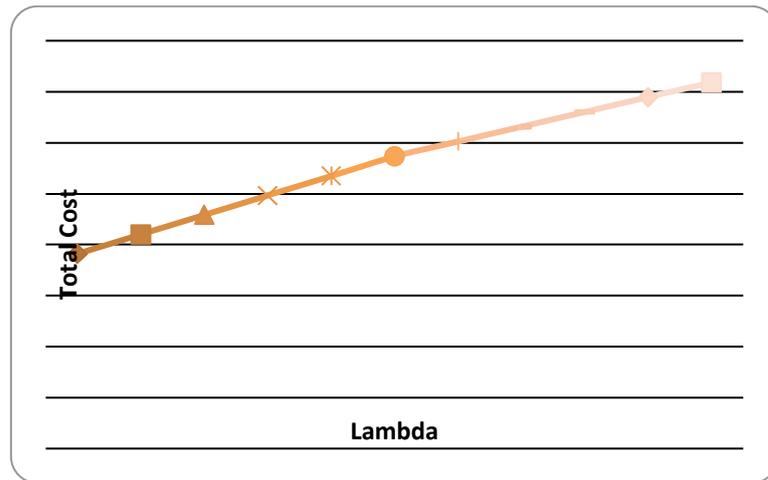


Figure 4. The Effect Of Changes In Λ On The Total Cost

4 CONCLUSIONS AND FUTURE WORKS

In this paper, a robust non-fixed MmTSP model for routing optimization of a 3PL provider is proposed. Our model captures the uncertainty in costs associated with moving from one node to another. To show the application of the model, a numerical test problem is solved and a set of sensitivity analyses are conducted to illustrate the effects of solution variance penalty parameter on different parts of the objective function. As our future work, we may extend the model to incorporate other decision criteria in addition to the routing decisions. Additional assumptions such as vehicle capacity, customer demands, multiple products and multiple-period planning horizon may close the gap between the model and the real-world problems. Considering uncertainty in parameters of extended model may be also a point of extension for our model.

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